# Hexahedron Point Mass Model and Teaching Learning Based Optimization for Balancing of Industrial Manipulators

Devi Singh Kumani, Himanshu Chaudhary

**Abstract**

Dynamic balancing of an industrial manipulator using hexahedron point mass model and teaching learning based optimization is presented in this paper. The minimization problem is formulated using concept of dynamically equivalent system of point-masses in hexahedron configuration for each link such that positive values for all point masses and link's inertias are ensured. To compute the shaking forces and moments the dynamic equations of motion for manipulator are systematically converted into the parameters of the equimomental point masses. The recently developed, 'Teaching Learning Based Optimization (TLBO)' is used to solve the optimization problem. The effectiveness of the methodology is demonstrated by applying it to a six-dof PUMA robot. shaking forces and moments at joints for the balanced and unbalanced PUMA manipulator are also provided to compare the result. The TLBO is a teaching-learning process inspired algorithm. It uses the mean value of the population and the best solution of the iteration to change the existing solution in the population, thereby improving the solution for the whole population and increasing the convergence rate. A MATLAB program is developed to find the design variables to minimize the shaking force and moment at the base of the robot. The objective function value obtained using TLBO is validated and compared with another population based solution i.e. GA and "fmincon". It is observed that TLBO is better than that of GA in terms of computational effort.

**Keywords:** Dynamic balancing, Teaching Learning Based Optimization (TLBO), Hexahedron point mass model, Shaking force and Shaking moment.

# 1 Introduction

The dynamic unbalance creates mechanical vibrations that induce noise, wear and fatigue etc. These are often undesired. A mechanism is called "Dynamically balanced", if no shaking forces and no shaking moment result at all. Balanced manipulators do not exert vibrations and can have both low cycle times and high accuracy. Thus it is essential to reduce the amplitude of vibration of the frame due to shaking forces and moments and also to smoothen out the highly fluctuating input torque to achieve the constant drive speed of the manipulator. The major disadvantage of existing dynamic balancing principles is that a considerable amount of mass and inertia is added to the system. Volkert van der Wijk et al., compared various dynamic balancing principles and stated that mass redistribution offers relatively lesser additional mass and additional inertia solution [1]. The shaking

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torques and forces at joints are to be considered in modelling of industrial manipulators. In robotic literature [2-3], lot of emphasis has been given to eliminate the shaking moments and forces from the equations of motion in order to perform inverse and forward dynamics needed for the control and simulation of the robot, respectively. Kochev, reviewed various methods for complete balancing of shaking moments of planer linkages, most of these methods are based on mass redistribution, addition of counter weights to moving linkages, and attachments of rotating discs [4]. Very few [5-7] have treated it as optimization problem with randomly generated population based solutions such as genetic algorithm for optimization.

For a given manipulator and its joint trajectories, the inertia-induced shaking moments and forces depend only upon the mass distribution of the moving links, i.e., the link masses, their mass centre locations and the inertias [8]. To minimize the shaking forces and moments, it is required to optimally distribute the masses of the links. This problem can be treated by the dynamically equivalent system of point masses, which is a convenient way to represent the inertia properties [9-10]. The dynamically equivalent system is also called equimomental system [11].

First, the links of a manipulator under study are represented by the equimomental system of point masses using hexahedron model with its CG coinciding with CG of link that ensures positive value for all point masses and offers practically implementable solutions. Chaudhary and Saha [10] developed the equations of motion in the parameters of point-mass that state the equivalence between the given system and the set of point-masses. These equations of motion are used for dynamic analysis. An optimization problem formulation is proposed to minimize the shaking forces/moments due to inertia forces at the joints of the industrial manipulator. The magnitudes of point masses are optimally distributed to reduce the inertia-induced forces and moments. This will minimize the shaking moments and forces at the joints of the manipulator. The Teaching Learning Based Optimization (TLBO) algorithm is used to solve optimization problem. This will minimize the shaking moments and forces at the joints of the manipulator apart from reducing the driving torque.

Rao et al. [13], proposed a novel method called "Teaching- learning-based optimization" (TLBO) for designing mechanical components. It does not require any algorithm parameter to be tuned, as required in genetic algorithm, thus makes the implementation simpler. TLBO uses the best solution of the current iteration to change the existing solution in the population, thereby increasing the convergence rate. This newly developed optimization algorithm that has been used for various optimization problems like constrained mechanical design optimization, multi-objective unconstrained and constrained functions, to solve complex bench mark functions and difficult engineering problems, discrete optimization of truss structures etc. [13-17]. Use of TLBO has been reported for optimizing process parameters as well [18-20]. So far no one has reported the use of the teaching-learning-based optimization (TLBO) technique for minimization of shaking forces, shaking moments in industrial manipulators.

# 2 Hexahedron Six Point Mass Model

To distribute link masses optimally, each link is treated as dynamically equivalent system of point masses. For this purpose a set of six point masses in hexahedron configuration as shown in Fig. 1, is defined. Body fixed frame $x_iy_iz_i$ is fixed to $i^{\text{th}}$

link at its CG and oriented such that $x_i$,$y_i$,$z_i$ become principal axes. It is assumed that five of the point masses, $m_{ij}$, are located at the vertices of a hexahedron. Subscripts i and j denote the $i^{th}$ link and its $j^{th}$ point mass, respectively. The sixth point mass is assumed to be located at CG of the link. The point masses are rigidly fixed to the frame $x_iy_iz_i$. The two systems, the rigid link and the system of six point-masses, are dynamically equivalent, if (i) the sum of all point-masses equal the mass of link (ii) the CG of the set of point masses coincides with the CG of rigid link, giving three conditions (iii) the moment of inertias and product of inertias for distributed point-masses is same as that of rigid link, giving six conditions.

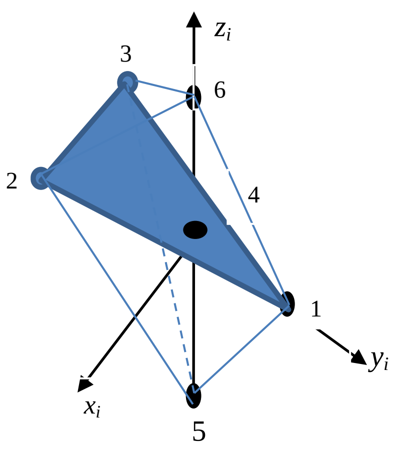


Fig. 1 Hexahedron model of six point-masses

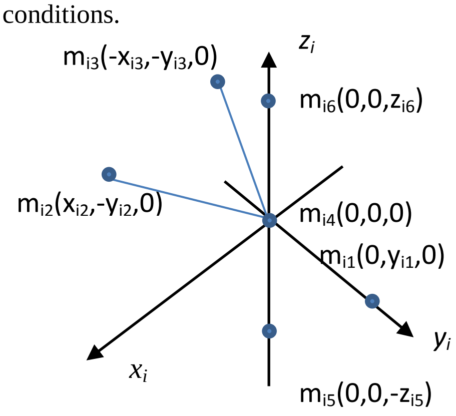


Fig.2 Coordinates of point masses

Assuming following relationship of point masses and their distances along axes $x_i$ , $y_i$ and $z_i$ , it makes CG of six point masses in hexahedron configuration to coincide with CG of link at the origin, we get the conditions known as equimomental conditions for $i^{th}$ link and are given as follows:

$$m_{i1} = 2m_{i2} = 2m_{i3} \ and \ m_{i5} = m_{i6} \quad (1)$$

$$x_{i2} = x_{i3} , y_{i1} = y_{i2} = y_{i3} \ and \ z_{i5} = z_{i6} \quad (2)$$

Where, mass, $m_i$, mass centre coordinates ($\bar{x}_i , \bar{y}_i , \bar{z}_i$), the moment of inertia ($I_{ixx}$, $I_{iyy}$, $I_{izz}$) and the product of inertia ($I_{ixy}$, $I_{iyz}$, $I_{izx}$) are defined for $i^{th}$ link. The coordinates ($x_{ij}$, $y_{ij}$, $z_{ij}$) of point mass $m_{ij}$ are defined for $j^{th}$ point mass of $i^{th}$ link.

Since all other coordinates are zeros for point masses 1,4,5 and 6 as shown in Fig. 2 and the location of masses 2 and 3 being symmetric, the product of inertias become zeros. So that $x_i$ , $y_i$ and $z_i$ are principal axes of the $i^{th}$ link and the mass centre is at the origin, i.e., $\bar{x}_i = \bar{y}_i = \bar{z}_i = 0$. This arrangement automatically satisfies the six equimomental conditions pertaining to centre of mass and product of inertias, the remaining four conditions of total mass and inertia about three axes gives :

$$m_i = 2m_{i1} + m_{i4} + 2m_{i5} \quad (3)$$

$$I_{ixx} = 2m_{i1}y_{i1}^2 + 2m_{i5}z_{i5}^2 \quad (4)$$

$$I_{iyy} = m_{i1}x_{i2}^2 + 2m_{i5}z_{i5}^2 \quad (5)$$

$$I_{izz} = m_{i1}x_{i2}^2 + 2m_{i1}y_{i1}^2 \quad (6)$$

Eqs. (3) – (6) contain 6 unknowns, three point masses $m_{i1}$, $m_{i4}$, $m_{i5}$, and three coordinates $x_{i2}$, $y_{i1}$, $z_{i5}$ . Assuming $m_{i1} = 2m_{i5} = \alpha\, m_i$ , Eqs. (3) – (6) gives :

$$m_{i4} = (1 - 3\alpha)m_i \quad (7)$$

$$x_{i2}^2 = (I_{iyy} + I_{izz} - I_{ixx})/2\alpha m_i \quad (8)$$

$$y_{i1}^2 = (I_{ixx} + I_{izz} - I_{iyy})/4\alpha m_i \tag{9}$$

$$z_{i5}^2 = (I_{ixx} + I_{iyy} - I_{izz})/2\alpha m_i \tag{10}$$

where, a constant α must satisfy, $1 > (1 - 3\alpha) > 0$, i.e. , $\alpha < 1/3$.

Finally ,knowing the mass and inertias of rigid links, Eqs. (7) – (10) provide unknown parameters of the point mass system. It does not contain negative point masses that being the problem with parallelepiped model in [9].

# 3. Optimization Problem Formulation

## 3.1 Optimality Criteria

The RMS value is preferred over other optimal criteria, as it gives equal emphasis on the results of every time instances of the cycle, and every harmonic component. In order to reduce shaking forces and moments at joints, the following objective function is proposed based on the RMS values:

$$f(\mathbf{x}) = \sum_{i=1}^{n} w_{i1} \tilde{f}_i^{\,c} + w_{i2} \tilde{n}_i^{\,c} \tag{11}$$

where $w_{i1}$ and $w_{i2}$ are the weighting factors whose values may vary depending on an application and preference order, whereas $\tilde{f}_i^c$ and $\tilde{n}_i^c$ are the RMS values of the shaking force, $f_i^c = \left|\mathbf{f}_i^c\right|$, and moment, $n_i^c = \left|\mathbf{n}_i^c\right|$, at the $i$th joint, respectively. We have taken both weighing factors as 1 so as to minimize the shaking force and the shaking moment both equally. The design variable vector $\mathbf{x}$ is defined in the next sub section.

## 3.2 Design Variables and Constraints

Based on the equations of motion given in [10], the shaking forces and moments are expressed as the function of the parameters of the point-masses. The point masses $m_{i1}$ ,…., $m_{i6}$, of each link are taken as the design variables. Note that the locations of the point-masses for each link are fixed in the link-fixed frame. For a manipulator having $n$ moving links, the 6$n$-vector of the design variables, $\mathbf{x}$, is then defined as

$$\mathbf{x} \equiv [\mathbf{m}_1^T, \ldots, \mathbf{m}_n^T]^T \tag{12}$$

where the 6-vector, $\mathbf{m}_i$ is as follows:

$$\mathbf{m}_{\text{i}} \equiv [m_{i1}\ m_{i2}\ m_{i3}\ m_{i4}\ m_{i5}\ m_{i6}]^T$$

In which $m_{ij}$ is as defined in six point mass hexahedron model. Considering $n$ links in manipulator, the problem of minimization of shaking forces and moments is finally stated as

$$\text{Minimize}\quad f(\mathbf{x}) = \sum_{i=1}^{n} w_{1i} \tilde{f}_i^{\,c} + w_{2i} \tilde{n}_i^{\,c} \tag{13a}$$

Subject to $m_i - \sum_{j=1}^{6} m_{ij} \leq 0$ for $i$ = 1, 2, 3…6 & $j \neq 1,4$ (13b)

$\beta\ m_i - m_{ij} \leq 0$ for $j$ =2, 3…6; $2\beta\ m_i - m_{ij} \leq 0$ for $j$ = 1, 4 (13c-d)

Where, a constant β is a fraction of point mass to link mass, say β =0.0001.The inequality (13b) ensures that the minimum link mass is $m_i$. Constraints (13c) & (13d) are imposed to keep point masses more than zero.

## 4. Teaching Learning Based Optimization (TLBO)

TLBO is a teaching-learning process inspired algorithm, proposed by Rao et al. [13]. There are two basic modes of learning by student, one through teacher (termed as teacher's phase) and other through interacting with fellow students (termed as learner's phase). The output in TLBO algorithm is considered as result or grade of the learners which depends on the quality of a teacher, who is considered as highly learned person. Interactive learning among colleagues also has impact on result. In TLBO, the group of learners is considered as population; different design variables are considered as different subjects offered to the learners and learners result is analogues to the 'fitness' value of the optimization problem. In the entire population the best solution is considered as teacher.

### 4.1 Teacher's Phase

In this phase, the teacher tries to increase the mean result of the class from any value M1 to his or her level. However, practically this is not possible and a teacher moves the mean of class from M1 to any other value M2 (M2 > M1), the difference between M2 and M1 depends on teacher's capability. The new value of design variable is obtained by adding the difference.

$X_{new,i} = X_{old,i} + r_i$ * ( difference between mean, $\overline{\mathbf{X}}$ and X of teacher ).

Where $r_i$ is random number between 0 to 1. If the objective function value for new solution is better than that of old, $f(X_{new,i}) < f(X_{\mathbf{old,i}})$ , new solution is accepted otherwise the old one is retained. This completes the, "Teacher's Phase".

### 4.2 Learner's Phase

In "Learner's Phase", learners increase their knowledge by interaction among themselves. A learner learns new things, if the other learner has more knowledge than him/her. The learner *i*, improves his/her knowledge through interaction with any other learner *j* from the population. The improvement is based on comparison of their objective function values as follows, for minimization problem:

$X_{new,i} = X_{old,i} + r_{il} * (X_{old,i} - X_{old,j})$, if $f(X_{old,i}) < f(X_{old,j})$ and

$X_{new,i} = X_{old,i} + r_{il} * (X_{old,j} - X_{old,i})$, if $f(X_{old,i}) > f(X_{old,j})$

Where $r_{il}$ is random number between 0 to 1. If the objective function value for new solution is better than that of old, $f(X_{new,i}) < f(X_{\mathbf{old,i}})$ , new solution is accepted otherwise the old one is retained. This process completes one cycle of iteration. The process from computation of variable mean $\overline{\mathbf{X}}$ onwards is repeated again, if the termination criteria remain unsatisfied. The design vector for which *f* (X) is minimum represents the optimal solution.

## 5 Application example

The six-dof PUMA robot is considered here to minimize shaking forces and moments given in [10] and proposed methodology developed in sections 4 and 5. The Denavit–Hartenberg (DH) parameters, link's masses, joint trajectory and inertias of the manipulator given in [12] are taken here for analysis and comparison purposes.

## 6 Results and Discussion

Since TLBO result is based on random population generated initially, therefore thirty trial runs were made with population size of 7200 (200*36) and with 20 iterations (generations) same as that used for GA to arrive at mean value of objective function at all six joints of the manipulator. The function values obtained in these trials are: **817.358**, **819.837**, **815.764**, **814.532**, **815.782**, **815.875**, **815.220**, **817.083**, **814.742**, **815.780**, **819.288**, **818.525**, **813.649**, **816.350**, **815.596**, **817.369**, **817.702**, **816.551**, **817.043**, **814.466**, **815.234**, **821.529**, **816.180**, **817.709**, **818.033**, **817.548**, **817.472**, **817.000**, **817.510**,and **819.608** giving mean value for optimized function using TLBO as **816.878** with standard deviation of **1.7297.** Similarly, the GA results with population of 7200, 20 generation and initial population as equimomental point masses for unbalanced PUMA gives objective function values as **829.474**, **827.834,** **828.763**, 823.028, **825.795, and 828.900.** The best value of objective function obtained in six trials of GA solution is **823.028**. Further, the value of objective function obtained using "fmincon" tool box of MATLAB is **817.32.** This is very near to the value obtained using TLBO.

The RMS values of the shaking forces and moments at each joint of the unbalanced (original), and optimized values obtained using TLBO and GA are given in Table 1. The shaking forces and moments values at joints 1to 6 pertains to mean objective function value of **817.000** and best objective function value of **823.021** for TLBO and GA respectively. To validate and compare the results, the same problem is solved using the "GA" algorithm of MATLAB Toolbox also. The objective function value obtained using TLBO is better than that obtained using GA **(817.000 < 823.021)**. The RMS values of shaking moments are reduced significantly at joints 1 to 3 through optimization as is evident from data in table 1. Sum of shaking moment at joints 1 to 6 is **170.16**, **32.87** and **38.90** N-m respectively for original (un-balanced), TLBO optimized and GA optimized PUMA robot. **Thus TLBO reduces the overall shaking moment further by nearly 3.5% vis-à-vis GA optimization.** This demonstrates that TLBO offers better solution than GA and also takes much lower computational time (~35 minutes) than GA(~16 hrs) in view of much lower functional evaluations . Figs. 3 and 4 shows that the optimization value converge faster in case of TLBO vis-à-vis GA. The RMS values of shaking force at joints 1 to 6 remains nearly same in all three cases as it depends on the total mass of the linkages which is kept same, we have redistributed the mass through optimization , which changes the inertia of link and reduces the shaking moment at joints of robot.

Table 1 The RMS values of shaking moments and forces

| PUMA ROBOT | RMS value of Shaking Moment (N-m) | | | | | | RMS value of Shaking Force (N) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Jt 1 | Jt 2 | Jt 3 | Jt 4 | Jt 5 | Jt 6 | Jt 1 | Jt 2 | Jt 3 | Jt 4 | Jt 5 | Jt 6 |
| Original (Unbalanced) | 73.659 | 76.173 | 14.704 | 5.447 | 0.105 | 0.076 | 367.975 | 264.790 | 110.219 | 24.138 | 13.797 | 3.453 |
| Optimized (TLBO) | 5.422 | 9.290 | 11.793 | 5.822 | 0.339 | 0.204 | 367.916 | 264.705 | 110.121 | 24.145 | 13.797 | 3.452 |
| % Reduction | 92.6% | 87.8% | 19.8% | ~same | ~same | ~ same | No significant reduction ( ~ 0 % ) | | | | | |
| Optimized (GA) | 5.102 | 26.812 | 2.663 | 4.132 | 0.112 | 0.076 | 367.919 | 264.709 | 110.128 | 24.123 | 13.798 | 3.453 |
| % Reduction | 93.1% | 64.8% | 81.9% | ~same | same | same | No significant reduction ( ~ 0 % ) | | | | | |

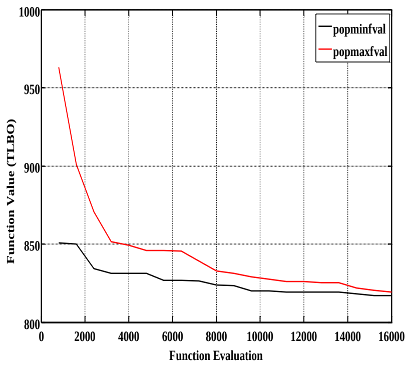


Fig. 3 Function evaluations TLBO

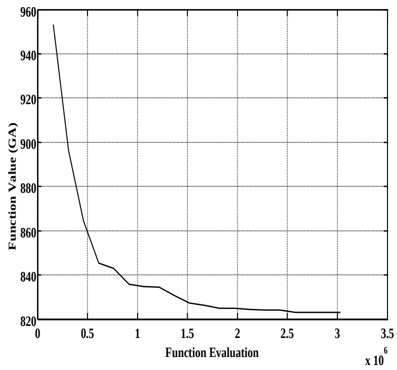


Fig. 4 Function evaluations GA

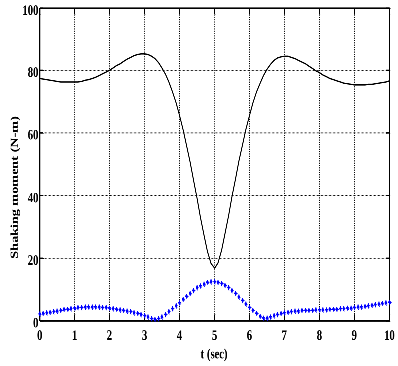


Fig. 5 Shaking moment at Joint 1

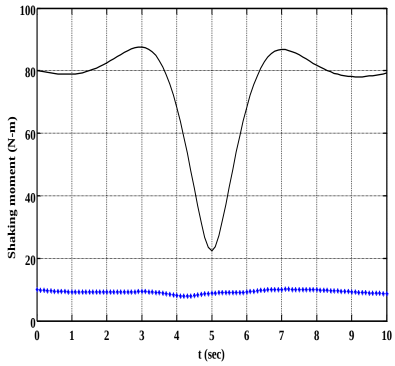


Fig. 6 Shaking moment at Joint 2

However, the peak value of shaking force is reduced as is demonstrated by figure 7. The Variation of shaking forces and shaking moments with respect to time for one complete cycle, for the original unbalanced and optimized manipulators are shown in Figs. 5, 6 and 7 respectively.

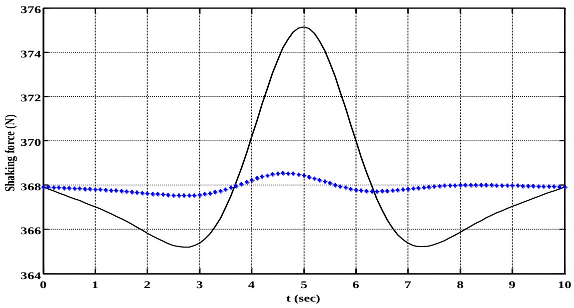


Fig. 7 Shaking force at Joint 1

## 8. Conclusion

The dynamic modelling of the manipulators is presented in terms of the equimomental system of point-masses using hexahedron model that ensures positive values for all point masses and eliminates non-linear constraints on link inertias. The optimization problem is solved using recently introduced algorithm TLBO. The results are compared and validated using the GA algorithm and “fmincon” of standard tool box in MATLAB. It is seen that the TLBO algorithm converges very fast with better optimization results as shown in Figs. 3 and 4 and Table 1. The hexahedron model provides the redistribution of the link masses such that the shaking moments and forces at joints are reduced to minimum, apart from providing positive values for all point masses and reducing the driving torques of the manipulator. Since the TLBO solution converges faster, it takes lesser computational time in comparison to GA. The results obtained using “fmincon” and TLBO are nearly same. Furthermore, the hexahedron model given solves the problem of negative masses faced with parallelepiped model.

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