Simulation of impact and rolling contact dynamics between a rigid body and a soft material using multibond graph approach

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Abstract

Impact is a contact between two bodies for a short duration. Dynamics of impact is quite complex as it involves application of contact forces for a short period. Evaluation of impact and rolling contact dynamics is important to understand many contact problems in robotics, manipulation tasks, multibody dynamics, explosive loading, etc. In this paper, impact and rolling contacts between a rigid sphere and a soft material are modeled using multibond graph. A specimen of silicon rubber, which is a soft material, is discretized into a number of eight nodes brick elements. Stiffness, mass and damping matrices of the soft material are calculated using finite element method, and used as **C**, **I** and **R** field respectively in the bond graph model. A contact algorithm is developed to detect dynamically contact location and contact area as contact interface changes during rolling and impact. Contact interface between the sphere and the soft material is assumed to be viscoelastic, and modeled using spring-damper subsystems along normal, tangential and bi-normal directions. Stick-slip friction between the two contacting surfaces is modeled using Kelvin-Voigt model. A rigid sphere is thrown on the soft material with some horizontal velocity, and it bounces many times and then rolls on the soft material before attaining a state of static equilibrium. The model determines contact area and spatiotemporal distribution of contact forces during impact and rolling contact. Dynamics of the soft material during compression and restitution along with the dynamics of the sphere is evaluated from the model.

Keywords: Impact, Contact dynamics, Bond graph, FEM, Viscoelastic

1 Introduction

Contact between two bodies may be a continuous contact or an impact contact. There is continuous contact during rolling or sliding of two contacting surfaces. Impact is a contact phenomenon between two colliding bodies. Impact occurs for short duration with sudden dissipation of energy. Dynamics of impact and rolling contact is important to understand many problems of robotics, manipulation tasks, multibody dynamics, astrophysics, explosive loading, press work, etc. Evaluation of dynamics

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of impact and rolling contact is quite complex and challenging because it involves dynamic change in contact area and distribution of contact forces. During impact, contact forces are applied over contact area for a short duration. It is important to calculate the instant of collision and contact location along with the distribution of contact forces. Impact contact can be modeled using impulse momentum principle. Dissipation of kinetic energy can be taken into account using coefficient of restitution. The approach does not calculate distribution of contact forces over the contact region [1]. Impact can also be modeled considering stress wave propagation [2]. Hertz theory augmented with damping to consider dissipation of energy is also used to determine force deformation relation that is used to calculate maximum indentation and period of impact [3-4]. Most of impact cases are neither perfectly elastic nor perfectly inelastic, and partial loss of kinetic energy is expressed using coefficient of restitution. The above approaches don't predict distribution of contact forces over contact area during a rigid-soft impact and rolling contact. Dynamics of impact and rolling contact has not yet been solved in systematic and algorithmic manner.

Bond graph is a graphical representation of dynamics of a physical system. The bond graph model is developed on the basis of power flow among various subsystems of a system [5-6]. Each bond shows clearly cause-effect relationship, and causality of bonds facilitates algorithmic derivation of first order states differential equations. The equations can be integrated numerically to evaluate dynamics of the physical systems.

Bond graph model of dynamics of soft contact interaction between a rigid body and a soft material has already been developed for planar case. The model was also validated experimentally of static case [7]. The model was applied to different geometries of the rigid bodies [8-10]. Bond graph approach has been used to solve different problems of contact mechanics. Merzouki *et al.* modeled dynamics of tyre-road interface using bond graph approach [11]. Bond graph is a unified approach and well suited to model systems from all energy domains.

In this work, contact model has been extended to spatial case. A specimen of silicon rubber is taken as a soft material. Stiffness, damping and inertia of the soft material affect impact dynamics. Stiffness and inertia matrices are calculated using finite element analysis and used as **C** and **I** fields in the bond graph model. A contact algorithm is developed to detect contact region and duration of contact. A rigid spherical ball is thrown with a horizontal force on the soft material underlay. It bounces on the soft material many times and rolls over it. Its translational and angular motions are stopped using proportional-derivative controllers, and it is allowed to settle in a state of static equilibrium. Contact interface between colliding rigid body and the soft material is assumed to be viscoelastic and modeled using spring-damper subsystems along tangent, normal and bi-normal directions. Stick-slip friction is considered between two surfaces. The model determines spatiotemporal distribution of contact forces over the contact area along with the dynamics of spherical ball and the soft material.

Calculation of **C** and **R** fields using finite element analysis is explained in section 2. Bond graph modeling of rigid body dynamics, soft material and contact interface is presented in section 3. MATLAB code is generated directly from the bond graph model and model is integrated numerically using ordinary differential equation (ODE) solver. The simulation results are presented in section 4. The work is concluded at the end of the paper in section 5.

2 Calculation of C and I Fields for the Soft Material

A specimen of silicon rubber of 0.2 *m* length, 0.04 *m* width and 0.03 *m* height is taken as a soft material. The material continuum is discretized into *N* number of eight nodes brick elements. An eight nodes hexahedron brick element is shown in Fig. 1.

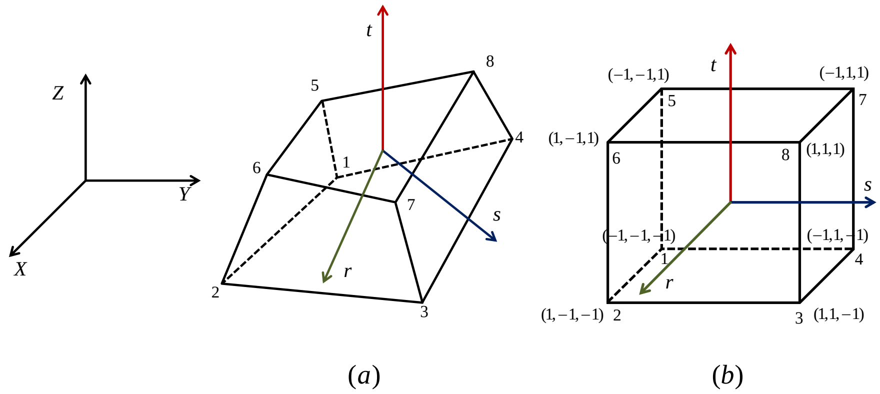


Figure 1: (a) Hexahedron element in Cartesian *X*, *Y*, *Z* coordinates. (b) Isoparametric hexahedron element in natural *r*, *s*, *t* coordinates.

Linear interpolation shape function of i^{th} node is given by Eq. (1). The value of N_i shape function is *1* at i^{th} node and *0* at the remaining nodes in an element.

$$N_i(r,s,t) = \frac{1}{8}(1+rr_i)(1+ss_i)(1+tt_i); \quad i = 1,2,3,........,8 \tag{1}$$

Each node is having three degree of freedom along *X*, *Y* and *Z* axes. Displacement field is given by Eq. (2).

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N_1 \; 0 \; 0 \; N_2 \; ....0 \\ 0 \; N_1 \; 0 \; 0 \; ....0 \\ 0 \; 0 \; N_1 \; 0 \; ....N_8 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ . \\ . \\ w_8 \end{Bmatrix} = [N]\boldsymbol{Q}_e \tag{2}$$

where $\boldsymbol{Q}_e$ is nodal displacement vector. Using the principle of virtual work, local stiffness matrix for each element is calculated [13-14] as given in Eq. (3)

$$K^e = \iiint_v [B]^T [D][B] \det[J] drdsdt \tag{3}$$

Where matrix $[B]$ contains derivatives of shape functions, $[D] \in \mathbb{R}^{6\times6}$ is a material matrix and $[J]$ is a Jacobian matrix. Local stiffness matrix is calculated by integrating Eq. (3) using six points Guassian quadrature rules. Local stiffness matrices are assembled into a global stiffness matrix $[K] \in \mathbb{R}^{3N\times3N}$ which is used as **C** field in the bond graph model of the soft material. Global inertia matrix $[I] \in \mathbb{R}^{3N\times3N}$ is also calculated using same shape functions and used as **I** field in the bond graph model. Damping of the soft material is considered to be constant and used as **R** field as $R\times\left([I] \in \mathbb{R}^{3\times3}\right)$. Bond graph models for dynamics of the soft material, rigid body dynamics are developed and explained in the next section.

3 Bond Graph Modeling

Bond graph models for rigid body dynamics, soft material and contact interface for planar case have already been developed and presented in [8-9]. In this work, the model is extended for spatial case of soft contact interaction. To make the paper self-sufficient, some repetition is required. The bond graph model for soft contact interaction is shown in Fig. 2. In the model, vector bonds with cardinality 3 are shown by thick half arrows, and scalar bonds are shown by thin half arrows.

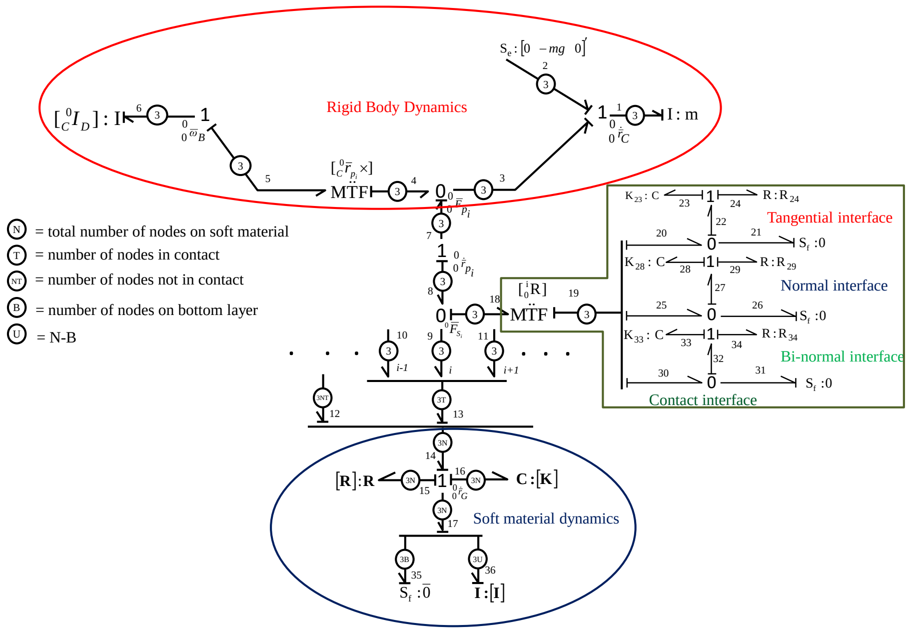


Figure 2: Bond graph model for spatial case of soft contact interaction.

A spherical ball is taken as a rigid body, and it is dropped on the soft material. It bounces many times before attaining state of static equilibrium. Velocity of any point P_i on the surface of the sphere is given as,

$$ {}_{0}^{0}\boldsymbol{r}_{\boldsymbol{P_i}} = {}_{0}^{0}\boldsymbol{r}_{\boldsymbol{C}} - \left[{}_{C}^{0}\boldsymbol{r}_{\boldsymbol{P_i}} \times \right] {}_{0}^{0}\boldsymbol{\omega}_{\boldsymbol{B}} \tag{4} $$

Where ${}_{0}^{0}\boldsymbol{r}_{C}$ is translational velocity and ${}_{0}^{0}\boldsymbol{\omega}_{\boldsymbol{B}}$ is angular velocity of the body observed and expressed in inertial frame. The bond graph model for the rigid body is developed on the basis of kinematics of any point P_i , and translational and rotational inertia are added to determine the dynamics. An algorithm to detect the contact location is developed. The position of each node on the upper layer of the soft material is determined with respect to the center of mass (CM) of the ball. The node which satisfies the constraint given by Eq. (5) contacts the ball.

$$ \left| {}_{C}^{0}\boldsymbol{r}_{S_i} \right| \leq \boldsymbol{Radius} \tag{5} $$

where ${}_{C}^{0}\boldsymbol{r}_{S_i}$ is position of S_i^{th} node from the CM of the ball. Contact between the ball and the soft material is modeled using penalty approach with additional dissipation. The S_i^{th} contact node is allowed to penetrate in the geometry of the spherical ball. Contact interface along normal direction is modeled using spring-damper subsystem. The restoring force directly proportional to the depth of penetration is generated, and further penetration is prevented. Unit vector normal to the point of contact P_i is determined as given in Eq.(6), and spring-damper subsystem is inserted between the contact point P_i on the ball and contact node S_i on the soft material along normal direction as shown in Fig. (3).

$$ {}_{S_i}^{0}\hat{\boldsymbol{r}}_{\boldsymbol{C}} = {}^{0}\hat{\boldsymbol{n}}_{\boldsymbol{i}} = \frac{{}_{S_i}^{0}\boldsymbol{r}_{\boldsymbol{C}}}{\left| {}_{S_i}^{0}\boldsymbol{r}_{\boldsymbol{C}} \right|} \tag{6} $$

Friction at the contact interface is considered to be stick-slip friction. Unit vector ${}^{0}\hat{\boldsymbol{t}}_{\boldsymbol{i}}$ along tangent to point of contact is considered along the direction of relative velocity of point P_i with respect to the S_i^{th} node. Tangential unit vector ${}^{0}\hat{\boldsymbol{t}}_{\boldsymbol{i}}$ is determined as given in Eq. (7).

$$ {}_{S_i}^{0}\hat{\dot{\boldsymbol{r}}}_{\boldsymbol{P_i}} = {}^{0}\hat{\boldsymbol{t}}_{\boldsymbol{i}} = \frac{{}_{0}^{0}\dot{\boldsymbol{r}}_{\boldsymbol{P_i}} - {}_{0}^{0}\dot{\boldsymbol{r}}_{\boldsymbol{S_i}}}{\left| {}_{0}^{0}\dot{\boldsymbol{r}}_{\boldsymbol{P_i}} - {}_{0}^{0}\dot{\boldsymbol{r}}_{\boldsymbol{S_i}} \right|} \tag{7} $$

where ${}_{S_i}^{0}\dot{\boldsymbol{r}}_{\boldsymbol{P_i}}$ is relative velocity of the point P_i with respect to the contact node S_i . Friction is modeled using spring-damper subsystem that is inserted between the point P_i and the node S_i along tangent direction as shown in Fig. (3). The node S_i remains attached with the point P_i within the range of static friction. Modeling of friction is done on the same lines as presented in [8-9]. Unit vector along bi-normal direction ${}^{0}\hat{\boldsymbol{b}}_{\boldsymbol{i}}$ is determined as given in Eq. (8).

$$ {}^{0}\hat{\boldsymbol{b}}_{\boldsymbol{i}} = {}^{0}\hat{\boldsymbol{n}}_{\boldsymbol{i}} \times {}^{0}\hat{\boldsymbol{t}}_{\boldsymbol{i}} \tag{8} $$

The orientation of i^{th} moving frame which is considered at the i^{th} contact point is given by $MTF : \left[{}_{i}^{0}R \right]$ as,

$$\left[{}^{0}_{i}\boldsymbol{R} \right] = \left[{}^{0}\hat{\boldsymbol{t}}_i \quad {}^{0}\hat{\boldsymbol{b}}_i \quad {}^{0}\hat{\boldsymbol{n}}_i \right] \tag{9}$$

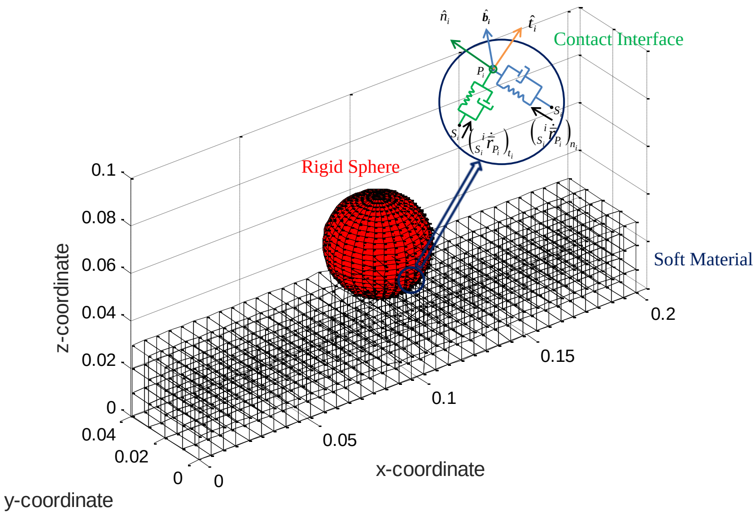


Figure 3: The rigid sphere rests on the soft material in state of static equilibrium is shown along with contact interface in tangent, normal and bi-normal directions.

Contact forces at each S_i^{th} contact node, on the sphere and total moment that acts on the sphere are determined using the same approach as presented in [8-9]. MATLAB code is generated from the bond graph model and system states equations are solved numerically using ordinary differential equation solver (ODE45) available. Simulation results for both impact and rolling contact are presented in the next section.

4 Simulation Results

The model is simulated for two cases: (1) the rigid sphere is dropped on the soft material; (2) the sphere is thrown on the soft material with some horizontal force. A sample of silicon rubber of length 0.2 *m*, 0.04 *m* width and 0.03 *m* height is taken as the soft material. It is discretized into $35 \times 4 \times 3$ brick elements along *x*, *y* and *z* axes respectively as shown in Fig. (3). For silicon, *Young's modulus of elasticity* 1.75971 *MPa* and density 1148 kg/m^3 is taken. The sphere of 0.02 *m* radius and 2 kg mass is taken. Impact contact is considered first as explained below.

Case (1): The sphere is dropped from a height of 0.08 *m* vertically above the inertial frame on the middle of the soft material as shown in Fig. (4). The soft material is deformed by 7.7 *mm* at the first impact. It bounces many times before attaining state of static equilibrium. Its height of bouncing reduces continuously due to dissipation of energy at each bounce, and it attains state of static equilibrium at the end. At the state of static equilibrium, the maximum deformation for the soft material is 2.42 *mm* as shown in Fig. (4). The position of the CM of the sphere when it is placed on the undeformed soft material is shown by red horizontal line.

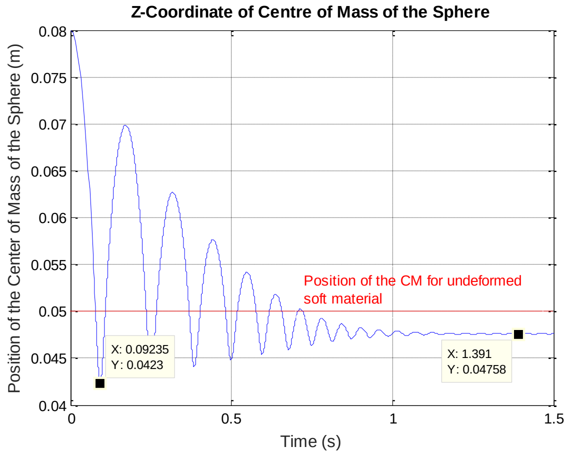
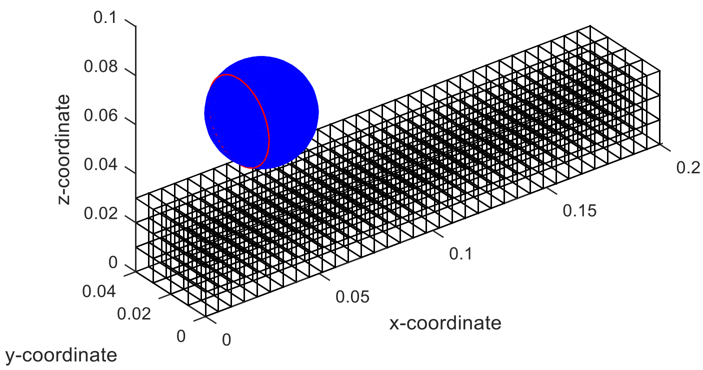


Figure 4: Z-coordinates of center of mass of the sphere. The soft material is deformed by 7.7 mm at the first impact.

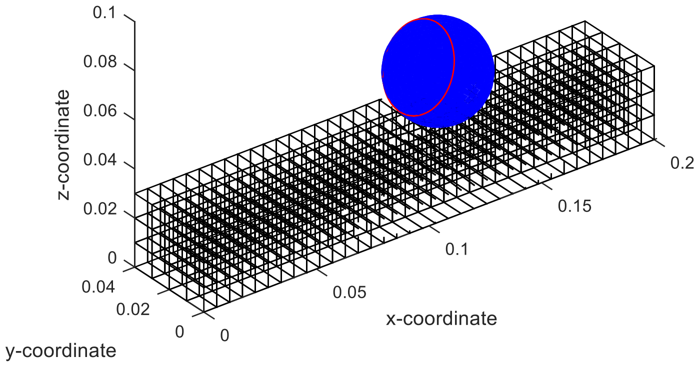
Case (2): The sphere is thrown on the soft material from a height of 0.01 *m* with a force of 5 *N* along horizontal direction. The horizontal force is applied for a short duration. It falls on the soft material and bounces many times, and then it starts rolling on the soft material. The position of sphere at two different instances of time is shown in Fig. (5).

The sphere is allowed to roll up to a distance of 0.1511 *m* on the soft material. Its translational motion along *X* and *Y* axes, and rotational motion is stopped using proportional-derivative (PD) controllers, and it is allowed to attain state of static equilibrium. *X*, *Y* and *Z* coordinates of the sphere is shown in Fig. (6). Z-coordinates indicate bouncing of the ball, and then resting on the soft material at the end.

The X-component of translational momentum and Y- component of the angular momentum the sphere is shown in Fig. (7). The PD controllers starts working at 1.5 *s*. The sphere takes approximately 2.5 *s* to be in state of static equilibrium. The model determines soft contact dynamics completely.



(a)



(b)

Figure 5: (a) Position of the sphere at time 0 *s*. (b) The position of the sphere at time 1.312 *s*. It rolls on the soft material as indicated by position of red circle on the periphery of the sphere.

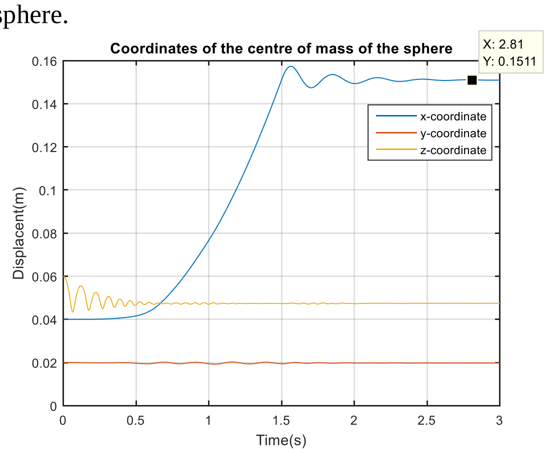


Figure 6: Coordinates of the CM of the sphere with respect to time.

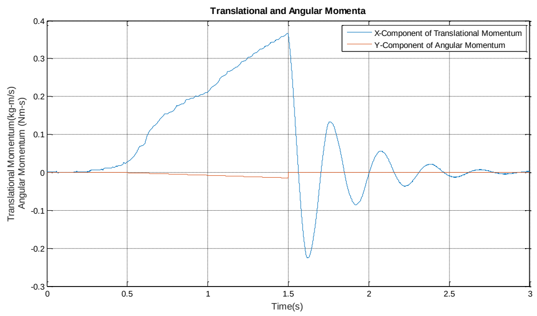


Figure 7: X-component of translational momentum and Y-component of angular momentum of the sphere.

5 Conclusions

Impact and rolling soft contact is modeled using multibond graph approach. The contact interaction between the sphere and the soft material in terms of efforts and flows is clearly understood. The model determines spatiotemporal contact location and contact forces during impact and rolling. The deformation of the soft material with restitution is obtained during simulation. The results clearly indicate the dissipation of energy during bouncing of the sphere. The bond graph approach is an algorithmic approach to evaluate contact dynamics. The model will be extended for manipulation of an object using soft fingers. The model is useful for development of anthropomorphic soft fingers for object manipulation in robotics hand, and understanding the tactile perception of human hand. The model will be used to develop control algorithm for simple manipulation task like holding and writing using a pencil.

6 References

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