Development of a Sun Tracking System using a 3-UPU Spherical Wrist Manipulator

Sudipta Pramanik, Ashitava Ghosal

Abstract

This paper presents the development of a sun tracking system using a 3-UPU parallel spherical "wrist" manipulator. The 3-UPU manipulator, consisting of a fixed base and an orient-able platform, is actuated by three linear motors at the prismatic (P) joints and has a special geometry which makes the 3-UPU manipulator equivalent to a spherical wrist with three rotational degrees of freedom. A flat mirror is mounted on the top of the platform and by actuating the prismatic joints, the platform can be made to track the sun and focus the incident energy at a distant receiver. The main advantage of the 3-UPU configuration is a result of the parallel nature – it can carry larger mirror (more loads) and has lower tracking error. The azimuth and the elevation angles of the sun change as the sun moves across the sky and are dependent on the date, time and location on the Earth's surface. A set of kinematic equations is derived which can be solved numerically to obtain the translations at the prismatic joints as the azimuth and the elevation angles of the sun change. Though the 3-UPU spherical wrist manipulator has its inherent singularity, the singularity condition is avoided by suitable design of the system and by judicious use of the workspace. The theoretical and numerical simulation results are validated by simulating the sun-tracking system with a CAD model.

Keywords: Sun tracking system, 3-UPU spherical wrist manipulator, Kinematics equations, CAD model.

1. Introduction

Incident solar energy on the earth can be converted to electricity by using large number of mirrors (also called heliostats) to focus the incident solar radiation on to a distant receiver situated on a tower where this thermal energy is transformed to electrical energy by a suitable heat engine [1, 2, 3]. A key requirement of such a heliostat is to track the sun throughout day and year in such a way that the incident energy is focussed on to the distant receiver. Since the sun moves in both the East-West and the North-South directions, a two-degrees-of-freedom mechanism is required to track the sun. There are two well-known serial configurations which are used to track the sun -- these are called as Azimuth-Elevation and Spinning-Elevation (also called Target-aligned) configurations [4, 5]. As the two degrees of freedom are in series, these configurations have the ability to resist limited wind load

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and to bear low self-weight, and as a result they incur high cost due to the heavy structural weight required to withstand required wind loading and self-weight.

It is well known that a pointing accuracy of 5 mrad or better is desired for a sun tracker [1]. To overcome the limitations of serial configuration sun tracker, a parallel 3-RPS manipulator has been proposed as a sun tracker [6]. The authors show that due to the parallel nature of the mechanism, the load carrying capability of the tracker is high and for a desired pointing accuracy, the structural weight is less. In the work by Massala [7], the author proposes several mechanisms, including 3-RPS parallel manipulator, for sun tracking. The kinematic equations for these proposals, however, are not available. Moreover, each of these proposals has one or more of the following limitations: non-optimum degree of freedom, bulky structure and hence high inertia, low load bearing capability, non-stationary centre of mass of platform.

In this paper, we propose a new parallel sun tracking mechanism which has some advantages over the 3-RPS parallel manipulator sun tracker. The proposed novel tracker is a parallel three degrees of freedom spherical “wrist’’ with three legs, -- each leg consisting of a universal (U), prismatic (P) and universal (U) joints with the prismatic (P) joint actuated. Since the manipulator has parallel configuration, it can carry larger load and requires less structural support material to achieve a desired pointing accuracy. Additionally, in the 3-RPS parallel manipulator sun tracker, the centre point of the mirror in the tracker shifts while tracking the sun and thus the tracker occupies more space in the heliostat field to prevent blockage losses. The 3-UPU spherical wrist manipulator sun tracker does not suffer from such limitation.

In this work, we develop the 3-UPU system with a flat mirror mounted on the top of a rotating platform to track the sun. We develop a set of equations which can be solved numerically to obtain the translations at the prismatic (P) joints such that the solar energy incident on the flat mirror can be focussed on to a distant receiver. The equations are derived based on the kinematics and geometry of the 3-UPU manipulator and on the laws of the optics. This paper is organized as follows: In section 2, we present the geometry of the 3-UPU manipulator using a CAD model. In section 3, we derive the equations used to obtain the translation of the prismatic joint for tracking the sun. We present the numerical results in section 4, conclusions and scope for future work in section 5.

2. The 3-UPU Spherical Wrist Manipulator

A 3D solid (CAD) model of a 3-UPU wrist manipulator, made using PRO/Engineer software [8], is shown in Fig. (1). One spatially orient-able platform is connected to a fixed base by three legs - each leg comprises of universal – prismatic – universal joints in that order and thus form a parallel manipulator. A flat mirror is mounted on the top of the platform which reflects the sun rays on to a distant receiver.

The degrees of freedom (dof) of the manipulator can be obtained by using the Grübler criterion [9]:

$$\mathrm{dof} = \lambda\left(\mathrm{N} - \mathrm{J} - 1\right) + \sum_{i=1}^{J} \mathrm{F_i} \tag{1}$$

where, $\lambda = 6$ for the spatial manipulator, $N = 8$, $\mathrm{J} = 9$, $\mathrm{F_i} = 2$ for a universal joint, $\mathrm{F_i} = 1$ for a prismatic joint. Thus, $\mathrm{dof} = 3$ and hence three joints need to be actuated. The three prismatic joints of the manipulator are chosen as actuated joints as they can be actuated easily using three independent linear motors.

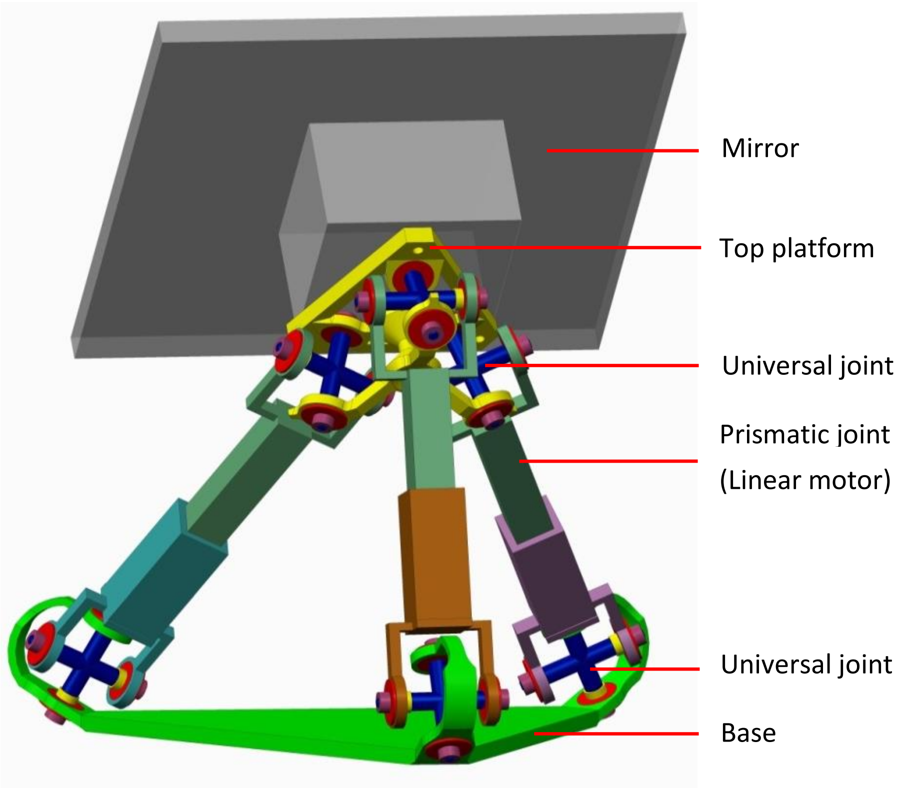

Figure 1: A solid (CAD) model of a parallel 3-UPU-wrist manipulator.

3. Kinematic Equations for a 3-UPU Spherical Wrist Manipulator as a Sun-Tracking System

Fig. (2) shows the kinematic diagram for the 3-UPU manipulator and a single UPU (‘i’th) leg of the manipulator. At any instant, parallel rays coming from the sun S falls on flat mirror at O and then get reflected to the receiver T. Three legs of the manipulator are 120˚ apart, as shown in the figure. Each universal joint is shown as two orthogonal revolute joints, representing two orthogonal axes of the universal joint. Two intermediate revolute joints of legs, one belonging to universal joint attached to the platform and another belonging to universal joint attached to the base are mutually parallel.

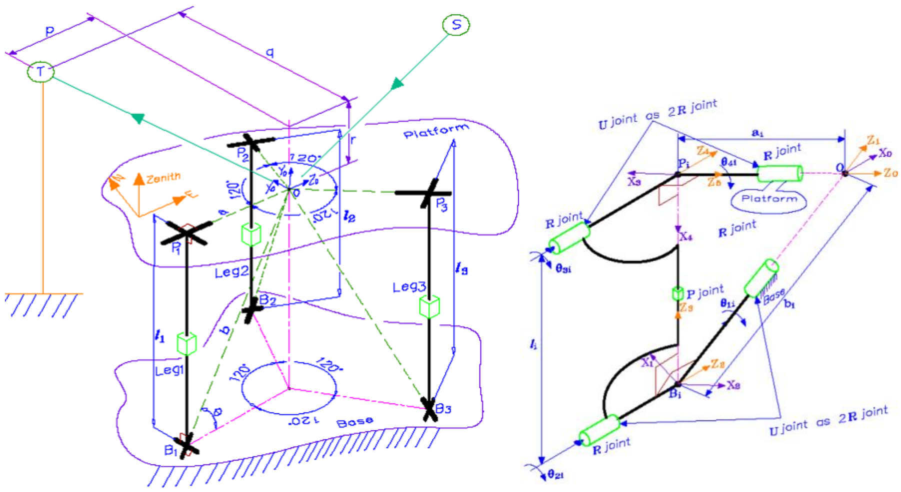

Figure 2: Schematic representation of a 3-UPU spherical wrist manipulator and the 'i'th leg, made using AutoCAD [10].

The Denavit-Hartenberg (D-H) parameters of a leg can be found using well-known approach (see for example [9]) and are given in Table 1 below.

Table 1: D-H table of a ('i'th) leg

Link (n= 1 to 5)	α_{n-1}	a_{n-1}	d_n	θ_n
1	$-\varphi$	0	$-b$	θ_{1i}
2	$\pi/2$	0	0	θ_{2i}
3	$\pi/2$	0	l_i	π
4	$\pi/2$	0	0	θ_{3i}
5	$\pi/2$	0	a	θ_{4i}

Using the D-H parameters, homogeneous transformation matrices ${}^{n-1}_{n}[T]$ are obtained and the position vector of point O (lying on P_1O link) with respect to the co-ordinate system $\boldsymbol{X_0}, \boldsymbol{Y_0}, \boldsymbol{Z_0}$ can be derived from the product ${}^{0}_{1}[T]\ {}^{1}_{2}[T]\ {}^{2}_{3}[T]\ {}^{3}_{4}[T]\ {}^{4}_{5}[T]$. This point is chosen to coincide with origin O, and we can obtain the following equations for the first leg.

$$-a\ \cos\theta_1 \sin(\theta_2+\theta_3) + l_1\ \cos\theta_1\ \sin\theta_2 = 0 \tag{2}$$

$$(-a\sin(\theta_2+\theta_3) + l_1\ \sin\theta_2)\sin\theta_1\cos\varphi + (a\cos(\theta_2+\theta_3) - l_1\cos\theta_2 - b)\sin\varphi = 0 \tag{3}$$

$$(a\sin(\theta_2+\theta_3) - l_1\ \sin\theta_2)\sin\theta_1\sin\varphi + (a\ \cos(\theta_2+\theta_3) - l_1\cos\theta_2 - b)\cos\varphi\ = 0 \tag{4}$$

where, $\theta_1 = \theta_{11}$, $\theta_2 = \theta_{21}$, $\theta_3 = \theta_{31}$. Similarly, the position vector of point O (lying on P_2O link) and the position vector of point O (lying on P_3O link) with respect to the co-ordinate system $\boldsymbol{X_0}, \boldsymbol{Y_0}, \boldsymbol{Z_0}$ can be derived for the second and third leg respectively and we get

$$-\frac{1}{2}(-a\ \sin(\theta_5+\theta_6)+l_2\ \sin\theta_5)\cos\theta_4+\frac{\sqrt{3}}{2}\sin\varphi\sin\theta_4\,(-a\sin(\theta_5+\theta_6)+l_2\ \sin\theta_5)-\frac{\sqrt{3}}{2}\cos\varphi\ (a\ \cos(\theta_5+\theta_6)-l_2\cos\theta_5-b)=0 \quad (5)$$

$$(-a\ \sin(\theta_5+\theta_6)+l_2\ \sin\theta_5)\sin\theta_4\cos\varphi+(a\cos(\theta_5+\theta_6)-l_2\cos\theta_5-b)\sin\varphi=0 \quad (6)$$

$$\frac{\sqrt{3}}{2}(-a\sin(\theta_5+\theta_6)+l_2\ \sin\theta_5)\cos\theta_4+\frac{1}{2}\sin\varphi\sin\theta_4\,(-a\ \sin(\theta_5+\theta_6)\ +l_2\ \sin\theta_5)-\frac{1}{2}\cos\varphi\,(a\cos(\theta_5+\theta_6)-l_2\cos\theta_5-b)=0 \quad (7)$$

$$-\frac{1}{2}(-a\ \ \sin(\theta_8+\theta_9)+l_3\ \sin\theta_8)\cos\theta_7-\frac{\sqrt{3}}{2}\sin\varphi\sin\theta_7\,(-a\ \ \sin(\theta_8+\theta_9)+l_3\sin\theta_8)+\frac{\sqrt{3}}{2}\cos\varphi\ (a\ \cos(\theta_8+\theta_9)-l_3\cos\theta_8-b)=0 \quad (8)$$

$$\cos\varphi\,(-a\ \sin\theta_7\sin(\theta_8+\theta_9)+l_3\ \sin\theta_7\ \sin\theta_8)+\sin\varphi\ (a\cos(\theta_8+\theta_9)-l_3\cos\theta_8-b)=0 \quad (9)$$

$$-\frac{\sqrt{3}}{2}(-a\ \cos\theta_7\ \sin(\theta_8+\theta_9)+l_3\ \cos\theta_7\ \sin\theta_8)+\frac{1}{2}\sin\varphi\,(-a\ \sin\theta_7\ \sin(\theta_8+\theta_9)+l_3\ \sin\theta_7\ \sin\theta_8)-\frac{1}{2}\cos\varphi\ (a\cos(\theta_8+\theta_9)-l_3\cos\theta_8-b)=0 \quad (10)$$

where, $\theta_4 = \theta_{12}$, $\theta_5 = \theta_{22}$, $\theta_6 = \theta_{32}$, $\theta_7 = \theta_{13}$, $\theta_8 = \theta_{23}$, $\theta_9 = \theta_{33}$.
It may be noted that none of the above equations contain θ_{4i} because of special choice of co-ordinate system and point O. Thus, nine equations, Eq.(2) through Eq.(10), involving nine passive joint variables and three active joint variables are obtained. Further, the position vector $\overrightarrow{\mathbf{0}_{\boldsymbol{P_1}}}$ of the point P_1 is obtained with respect to the co-ordinate system $\boldsymbol{X_0}, \boldsymbol{Y_0}, \boldsymbol{Z_0}$ by the products ${}^0_1[T]\ {}^1_2[T]\ {}^2_3[T]$. Similarly, the position vectors $\overrightarrow{\mathbf{0}_{\boldsymbol{P_2}}}$ and $\overrightarrow{\mathbf{0}_{\boldsymbol{P_3}}}$ with respect to the co-ordinate system $\boldsymbol{X_0}, \boldsymbol{Y_0}, \boldsymbol{Z_0}$ can also be found out. Thus,

$$\overrightarrow{\mathbf{0}_{\boldsymbol{P_1}}}=[p_{x1}\quad p_{y1}\quad p_{z1}]^T,\ \overrightarrow{\mathbf{0}_{\boldsymbol{P_2}}}=[p_{x2}\quad p_{y2}\quad p_{z2}]^T\ \&\ \overrightarrow{\mathbf{0}_{\boldsymbol{P_3}}}=[p_{x3}\quad p_{y3}\quad p_{z3}]^T \quad (11)$$

where,

$$p_{x1} = l_1\ \cos\theta_1\ \sin\theta_2 \quad (12)$$

$$p_{y1} = l_1\ \sin\theta_1\ \sin\theta_2\ \cos\varphi-(l_1\ \cos\theta_2+b)\sin\varphi \quad (13)$$

$$p_{z1} = -l_1\ \sin\theta_1\ \sin\theta_2\ \sin\varphi-(l_1\ \cos\theta_2+b)\cos\varphi \quad (14)$$

$$p_{x2} = (-\tfrac{1}{2} l_2 \cos\theta_4 + \tfrac{\sqrt{3}}{2} l_2 \sin\varphi \sin\theta_4) \sin\theta_5 + \tfrac{\sqrt{3}}{2} \cos\varphi \ (l_2 \cos\theta_5 + b) \qquad (15)$$

$$p_{y2} = l_2 \sin\theta_4 \sin\theta_5 \cos\varphi - (l_2 \cos\theta_5 + b) \sin\varphi \qquad (16)$$

$$p_{z2} = (\tfrac{\sqrt{3}}{2} l_2 \cos\theta_4 + \tfrac{1}{2} l_2 \sin\varphi \sin\theta_4) \sin\theta_5 + \tfrac{1}{2} \cos\varphi \ (l_2 \cos\theta_5 + b) \qquad (17)$$

$$p_{x3} = (-\tfrac{1}{2} l_3 \cos\theta_7 - \tfrac{\sqrt{3}}{2} l_3 \sin\varphi \sin\theta_7) \sin\theta_8 - \tfrac{\sqrt{3}}{2} \cos\varphi \ (l_3 \cos\theta_8 + b) \qquad (18)$$

$$p_{y3} = l_3 \sin\theta_7 \sin\theta_8 \cos\varphi - (l_3 \cos\theta_8 + b) \sin\varphi \qquad (19)$$

$$p_{z3} = (-\tfrac{\sqrt{3}}{2} l_3 \cos\theta_7 + \tfrac{1}{2} l_3 \sin\varphi \sin\theta_7) \sin\theta_8 + \tfrac{1}{2} \cos\varphi \ (l_3 \cos\theta_8 + b) \qquad (20)$$

Since, the points P_1, P_2 and P_3 are fixed on the platform, the distance between any two of these points remains constant. We choose $P_1P_2 = P_2P_3 = P_3P_1 = L$ (say). Thus, we get three equations

$$(p_{x1} - p_{x2})^2 + (p_{y1} - p_{y2})^2 + (p_{z1} - p_{z2})^2 = L^2 \qquad (21A)$$
$$(p_{x2} - p_{x3})^2 + (p_{y2} - p_{y3})^2 + (p_{z2} - p_{z3})^2 = L^2 \qquad (21B)$$
$$(p_{x3} - p_{x1})^2 + (p_{y3} - p_{y1})^2 + (p_{z3} - p_{z1})^2 = L^2 \qquad (21C)$$

Finally, we get two equations from the specular light reflection laws. Equating the angle of incidence to the angle of reflection, we get

$$\cos^{-1}\big((q\,x + r\,y + p\,z)/(\sqrt{p^2 + q^2 + r^2}\,\sqrt{x^2 + y^2 + z^2})\big) =$$
$$\cos^{-1}\big((x \cos\alpha \cos A + y \sin\alpha + z \cos\alpha \sin A)/(\sqrt{x^2 + y^2 + z^2})\big) \qquad (22)$$

And since the incident ray, the reflected ray and mirror normal are co-planer, we get

$$x(p \sin\alpha - r \cos\alpha \sin A) + y(q \cos\alpha \sin A - p \cos\alpha \cos A) + z(r \cos\alpha \cos A - q \sin\alpha) = 0 \qquad (23)$$

where, $(x, y, z)^T$ is the mirror normal vector and is obtained by $\left(\overrightarrow{\mathbf{0}_{P_1}} - \overrightarrow{\mathbf{0}_{P_c}}\right) \times \left(\overrightarrow{\mathbf{0}_{P_3}} - \overrightarrow{\mathbf{0}_{P_c}}\right)$ and $\overrightarrow{\mathbf{0}_{P_c}} = \left(\overrightarrow{\mathbf{0}_{P_1}} + \overrightarrow{\mathbf{0}_{P_2}} + \overrightarrow{\mathbf{0}_{P_3}}\right)/3$.

It may be noticed that Eq.(22) and Eq.(23) contain parameters α (elevation angle of the sun) and A (azimuth angle of the sun) which are dependent on time, date and sun tracking system installation location. These can be found out at any time, date and sun tracking system location [11]. The quantities a, b, L, φ for a tracker is known by its design and p, q and r are the distances of receiver from point O along local geographical East, North and Zenith directions, respectively and are known. Hence, the twelve unknowns i.e. θ_1 through θ_9 and l_1 through l_3 are determined by solving the twelve equations (Eq.(2) through Eq.(10) and Eq.(21) through Eq.(23)), numerically. It is important to note that values of x, y and z must ensure each inverse cosine terms of both the sides of Eq. (22) have value less than 90˚. This is essential to make sure that mirror normal position vector is oriented towards the side where the sun and the receiver are situated and hence reflecting mirror surface faces the sun always. Finally, once the actuated joint variables l_1, l_2 and l_3 are determined, they are actuated and thus the mirror can be made to track the sun.

3.1. Singularity of the 3-UPU-Wrist Manipulator Sun-Tracking System

It is known that the 3-UPU wrist manipulator suffers from singularity condition. At a singularity, the manipulator gains extra degree-of-freedom and orienting the mirror in singularity condition becomes difficult. Hence, it is important to know the singularity condition so that the manipulator can be controlled to avoid the singularity. It can be shown that when the three revolute joint axes corresponding to the joint variables θ_2, θ_5 and θ_8 are coplanar and the plane is orthogonal to its leg axes, singularity appears [12]. The sun-tracking system is designed and operated in such a way that the actuated joint variables l_1, l_2 and l_3 never results in the 3-UPU manipulator to reach a singularity.

4. Simulation Result

Algorithm for finding the azimuth and elevation angle of the sun with respect to any place on earth is readily available [11]. In Fig.(3), the azimuth and elevation angle of the sun on 8th March 2015 is plotted using MATLAB [13] for Bangalore having 12˚58'13" latitude and 77˚33'37" longitude.

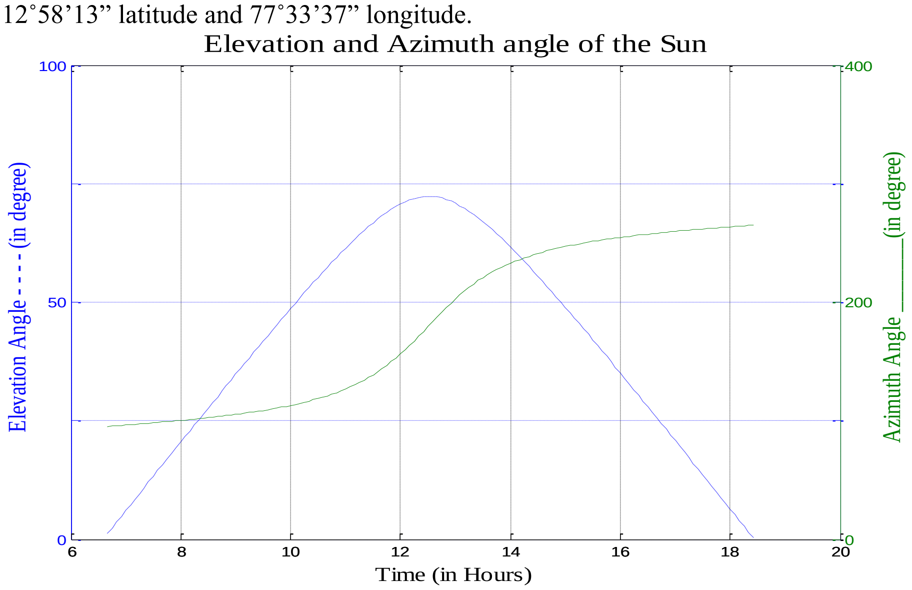

Figure 3: Plot of Azimuth and Elevation angle of Sun on 8th March 2015 at 12˚58'13" latitude and 77˚33'37" longitude

It may be noted that azimuth angle is measured positive clockwise and assumes value 0° at local geographical North direction in the horizon plane, i.e., on horizon plane, azimuth angle is positive while measured from North and towards East. It can take the value between 0° and 360°. The elevation angle is measured positive from horizon and towards local zenith direction (vertically up) in local vertical plane (plane contains the sun), i.e., on local vertical plane, elevation angle is 0° at horizon direction and 90° at zenith direction. It can take the value between -90° and +90°.

The manipulator design was done considering assembly, fabrication and workspace point of view. Following is a set of design parameters that satisfy the said criteria: $a = 86.9873$ mm, $b = 220$ mm, $p = 100$ m, $q = 80$ m, $r = 10$ m, $\varphi = 60°$, $L = 51.5309$ mm. Twelve joint variable values are obtained solving the twelve equations numerically using Matlab in order to track the sun typically on 8th March 2015 for Bangalore with the values of azimuth and elevation angles taken from the graph in Fig.(3). Fig.(4) shows the plots of prismatic joint l_1, l_2 and l_3 vs. time. The values obtained from numerical solution of the twelve equations were validated through a solid (CAD) model of the manipulator using PRO/Engineer mechanism simulation software [8]. Similar plots can be obtained for any other day and any other locations on the surface of Earth.

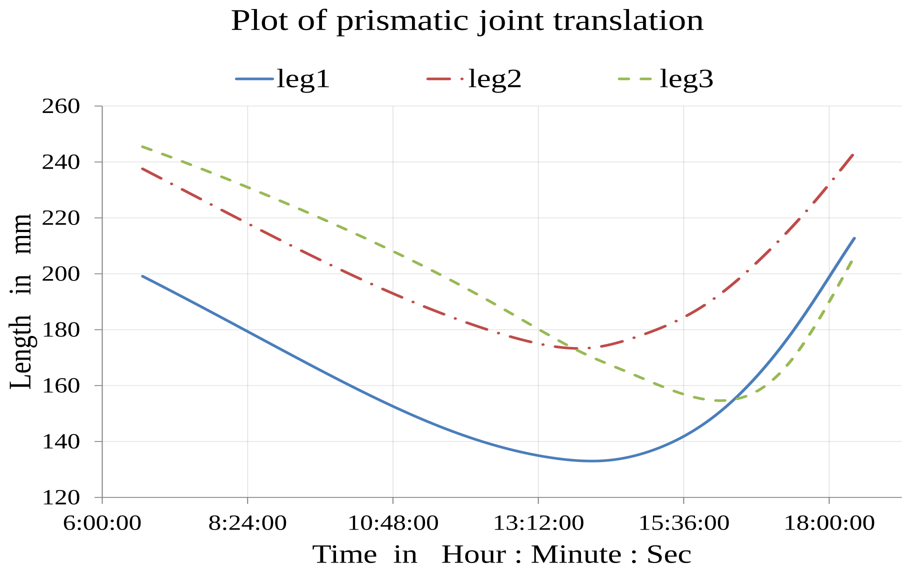

Figure 4: Plot of prismatic joints vs. time in GMT for 12˚58'13" latitude and 77˚33'37" longitude on 8th March 2015.

5. Conclusion

This paper deals with the use of a novel 3-UPU spherical wrist manipulator as a sun tracking system. Twelve kinematic equations modelling the tracking of the sun are derived and solved numerically. The values of the prismatic joint variables obtained from numerical simulations are validated through a CAD solid model of the manipulator and by actuating the joint variables. In future work, we plan to fabricate the proposed 3-UPU spherical wrist parallel manipulator and validate the algorithms developed here, experimentally.

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