SYSTEM IDENTIFICATION: A STUDY OF VARIOUS METHODS FOR CONTINUOUS SYSTEMS

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Abstract

This paper addresses and evaluates the methods of system identification of a continuous system. These methods help in finding out the system properties i.e. natural frequency and damping. The natural frequency and damping of a system provides the information about the resonance condition, which is of utmost importance in systems where failure can happen due to natural frequency excitation like road bumps exciting the car chassis, waves hitting an offshore.

1. INTRODUCTION

The vibration analysis methods, used to study and evaluate the system parameters are Fast Fourier Transformation (FFT) and Frequency Response Function (FRF). FFT helps to convert the time domain into a frequency domain hence making the data more compatible to analyze.

For system identification an impulse is given to the system using an impact hammer. This impact is applied for an infinitesimal time duration exciting all the frequencies over a large range. The first mode that gets excited in the frequency spectrum is the fundamental natural frequency of the system. This method is applicable to the case when the system to be tested is small.

When the system is relatively larger, shaker modal testing is used. The shaker excites the system according to the amplified input signal it receives. The most commonly used input signals for modal testing are the sine sweep and random frequency profile

2. MATERIALS AND METHODS

2.1. Impact Test

The impact test is a simple test that requires little hardware and provides shorter measuring time. A hammer (Fig. 1) or other impact device equipped with a force transducer is used to strike the structure and an accelerometer measures the structure's response vibration.

Impact hammers are available in weights varying from a few ounces to several pounds. Also, mass can be added to or removed from most hammers, making them useful for testing objects of varying sizes and weights. The frequency content of the

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energy applied to the structure is a function of the stiffness of the contacting surfaces and, to a lesser extent, the mass of the hammer. The stiffness of the contacting surfaces affects the shape of the force pulse, which in turn determines the frequency content.

Fig. 1. IMPACT HAMMER

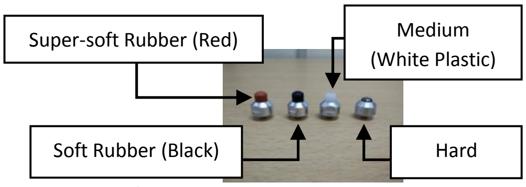


Fig. 2.DIFFERENT IMPACT HAMMER TIPS

It is not feasible to change the stiffness of the test object; therefore the frequency content is controlled by varying the stiffness of the hammer tip. The harder the tip, the shorter is the pulse duration and thus higher is the frequency content. Figure 2 shows impact tips with different stiffness. The effect of impact tip stiffness on the force spectrum is illustrated in Fig. 3 and Fig. 4. A disadvantage to note here is that the force spectrum of an impact excitation cannot be band-limited at lower frequencies when making zoom measurements, so the lower out-of-band modes will still be excited.

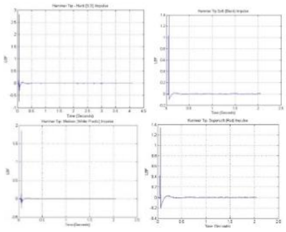


Fig. 3. IMPULSE GRAPH OF DIFFERENT HAMMER TIPS

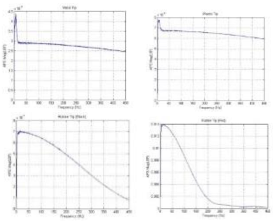


Fig. 4. APS OF DIFFERENT HAMMER TIP IMPULSE

Force and response windows are used for this specific application - computing the transfer function of a mechanical structure using an impulsive force excitation (Fig. 3). The response signal is an exponential decaying function and may decay out before or after the end of the measurement. The window designed to accomplish either result, called the exponential window, is shown for response of each tip in Fig. 5. [1]

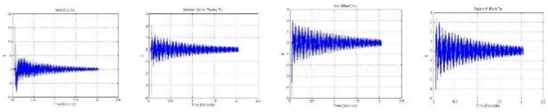


Fig. 5. RESULT OF EXPONENTIAL WINDOW IN RESPONSE

2.2. Shaker Test

The most useful shakers for modal testing are the electromagnetic, often called electro-dynamic (shown in Fig. 6) and the electro-hydraulic or hydraulic type. With the electromagnetic shaker, the more common of the two, force is generated by an alternating current that drives a magnetic coil.

There are several potential problem areas to consider when using a shaker system for excitation. To begin with, the shaker is physically mounted to the structure via the force transducer, thus creating the possibility of altering the dynamics of the structure. With lightweight structures, the mechanism used to mount the load cell may add appreciable mass to the structure. This causes the force measured by the load cell to be greater than the force actually applied to the structure.

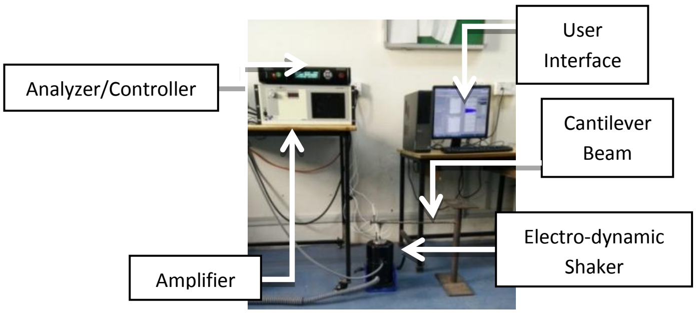


Fig. 6. SHAKER TEST SETUP

Figure 7 describes how this mass loading alters the input force. Since the extra mass is between the load cell and the structure the load cell senses this extra mass as part of the structure. Since the frequency response is a single input function, the shaker should transmit only one component of force in line with the main axis of the load cell. In practical situations, when a structure is displaced along a linear axis it also tends to rotate about the other two axes. To minimize the problem of forces being applied in other directions, the shaker should be connected to the load cell through a slender rod, called a stinger, to allow the structure to move freely in the other directions. This rod, shown in Fig. 8, has a strong axial stiffness, but weak bending and shear stiffness. In effect, it acts like a truss member, carrying only axial loads but no moments or shear loads. [1]

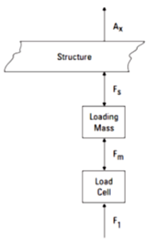


Fig. 7. MASS LOADING OF SHAKER

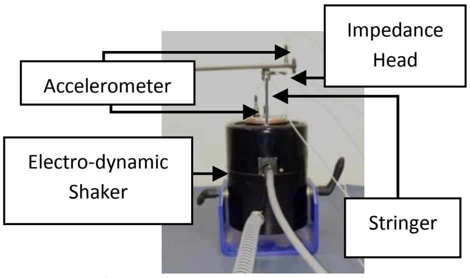


Fig. 8. STRINGER ATTACHMENT TO BEAM

$$\boldsymbol{F_S = F_M - M_M A_X} \quad \textbf{...}(2)$$

F_S = Actual force
F_M = Force measured by load cell
M_M = Loading mass
A_X = Acceleration

Another potential problem associated with electromagnetic shakers is the impedance mismatch that can exist between the structure and the shaker coil. The electrical impedance of the shaker varies with the amplitude of motion of the coil. At a resonance with a small effective mass, very little force is required to produce a response. This can result in a drop in the force spectrum in the vicinity of the resonance, causing the force measurement to be susceptible to noise. Figure 9 illustrates an example of this phenomenon.

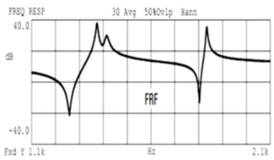


Fig. 9. SHAKER/STRUCTURE IMPEDENCE MISMATCH

2.3. Acoustic Test

A sound level meter (SLM) is a device used to make frequency-weighted sound pressure level measurements displayed in dB-SPL.

All SLMs feature an omni-directional measurement quality condenser microphone, a mic preamp, frequency weighting networks, an RMS detector circuit, averaging circuits, the meter display, AC and DC outputs used to feed other measurement devices or for recording (Fig. 10). [2]

Because we humans do not hear frequencies in a linear manner (as illustrated in the often referred to Fletcher-Munson curves, aka: equal loudness contours), sound level measurements made with a flat response do not accurately reflect how we perceive sound.

The most common weighting that is used in noise measurement is A-Weighting (Fig. 11).

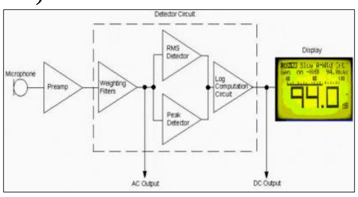


Fig. 10. FUNCTIONAL DRAWING OF SLM

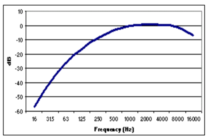


Fig.11.A-WEIGHTED

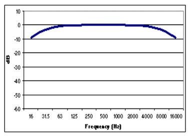


Fig.12. C-WEIGHTED

Although the A-Weighted response is used for most applications, C-Weighting is also available on many sound level meters. C Weighting is usually used for Peak measurements and also in some entertainment noise measurement, where the transmission of bass noise can be a problem. C-weighted measurements are expressed as dBC or dB(C). Z-weighting is a flat frequency response of 10Hz to 20 kHz ±1.5dB.

3. EXPERIMENTAL SETUP

For the study presented here, impact tests, shaker tests and vibro-acoustics tests were conducted on a cantilever beam. The experimental setup for the impact test is shown in the Fig. 13. An impact hammer containing a load cell of sensitivity 10 mV/lbf is used to excite the natural frequencies of the cantilever beam. The vibration signatures obtained as a result of the impact is measured using three piezoelectric accelerometers. One accelerometer of sensitivity 102.2 mV/g, is mounted on the free end of the cantilever beam, another accelerometer of sensitivity 101.3 mV/g, is placed at 1/3rd distance from the free end and the third accelerometer of sensitivity of 101.8 mV/g, is placed at 2/3rd distance from the free end. The number of meshed points denote the degree of freedom of the system. These reference points are selected such that they are not located at the node of a particular mode otherwise the mode will be suppressed. Exponential average of 8 impact responses is considered for more accurate results. As the output of the accelerometer is of low level and contains some unwanted frequencies, some form of pre-processing is required before analyzing the data. A 4-input, 1-output vibration analyzer system [3] (Spider-81 Vibration Controller System) amplifies the output data of the accelerometer and converts the data into frequency domain using Electronic Data Management (EDM) software. The controller shown in the Fig. 6 consists of the Preamplifier and Spectrum Analyzer.

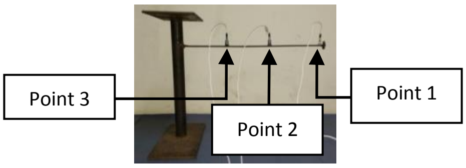


Fig. 13. EXPERIMENTAL SETUP FOR IMPACT TEST

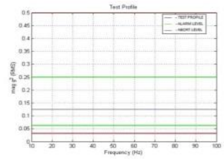


Fig. 14. SHAKER TEST PROFILE

The experimental setup for shaker test is shown in Fig. 8. In this test the cantilever beam is excited by an electro-dynamic shaker [3] with the help of a stinger. A control accelerometer of sensitivity 101.3 mV/g, is placed on the armature of the shaker. An impedance head (force sensitivity 98.73 mV/lbf and acceleration sensitivity 101.3 mV/g) is attached to the stinger. Another accelerometer of sensitivity 102.2 mV/g, is mounted on the free end of the cantilever beam to measure the response. A Chirp (swept sine) signal [4] is synthesized in a vibration analyzer system [5] (Spider-81 Vibration Controller System), as input to the system. Figure 14 shows the test profile of the chirp signal. The swept sine test allows for higher RMS input loads and as a result it leads to a much cleaner modal responses frequently. The swept sine test is also used because it provides symmetrical excitation to excite the symmetrical modes. The amplifier [6] transfers the power from the line source and transforms it on electrical signals of desired frequency and amplitude. The vibration analyzer system monitors the test, the output exciter acceleration and supplies the adequate input signal for the amplifier.

The acoustic signatures are obtained using a sound level meter (SLM), shown in Fig. 15 [7]. A mesh is designed around the cantilever beam and the measurement point is selected based on the maximum likelihood localization method [6]. The SLM is kept at the prescribed distance of 5 cm from the free end of the cantilever beam. The readings are taken in the absence of any external noise field in a studio room. The setup for acoustic measurements is shown in Fig. 16.

Fig. 15. SOUND LEVEL METER (SLM)

Fig. 16. SETUP FOR ACOUSTIC TEST

All acoustic measurements are taken at the cantilever beam plane's height. This height is chosen based on the maximum likelihood principle, according to which the sound intensity at all grid points are not the same and it varies approximately according to the Inverse square law [8, 9] as shown in Eq. (3):

$$Intensity \propto \frac{1}{Distance^2} \quad (3)$$

From Eq. (3), it can be inferred that the maximum sound intensity from the cantilever beam would most likely be detected at the point 1 shown in Fig. 13.

4. NUMERICAL APPROACH

The continuous systems considered were analyzed using various theoretical and analytical methods.

4.1. Euler's Equation for Beams

The natural frequencies of the cantilever beam are obtained by the Euler's equation, Eq. (4), for beams [10].

$$\omega_n = ß_n^2 \sqrt{\frac{EI}{\rho}} = (ß_n l)^2 \sqrt{\frac{EI}{\rho l^4}} \quad (4)$$

The first five natural frequencies are found using the dimensions and material properties of the cantilever beam under consideration and $ß_n l$ as shown in Table 1.

Table 1. FIRST FOUR MODES FROM EULER'S METHOD

Mode	$ß_n l$	f_n (Hz)
1	1.875	14.0985
2	4.695	88.3976
3	7.855	247.4313
4	10.9955	484.8412

4.2. FEM Analysis

On performing the Finite Element Method (FEM) analysis using MATLAB [11], the mode shapes and corresponding natural frequencies, as shown in Fig. 17, were obtained. Also a rectangular beam with the dimensions that of the cantilever beam considered is modeled in ANSYS. The element type of SOLID185, with a global mesh size of 0.001 is used. All degrees of freedom of one end of the beam are constrained and all the other nodes are constrained only along the x-direction which is along the breadth of the beam to eliminate torsional modes.

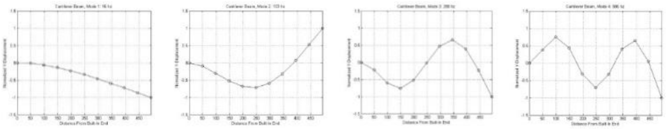


Fig. 17. FEM ANALYSIS RESULT FROM MATLAB

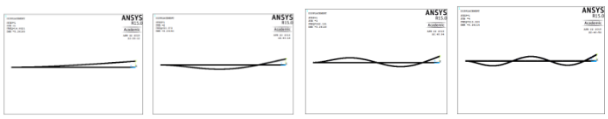


Fig. 18. FEM ANALYSIS RESULT FROM ANSYS

The results from the FEM analysis are tabulated in Table 2.

5. EXPERIMENTAL RESULTS

5.1. Impact Test Results

The impact test is carried out following the experimental procedure as illustrated in Section 3. Figure 19 and 20 shows the magnitude, phase, real and imaginary part of FRF. The impact is given at the free end of the cantilever beam with the accelerometer measuring the response at the same point. This is a special measurement referred to as the drive point measurement. The amplitude peaks in the FRF graph gives us all the natural frequencies of the cantilever beam in the particular frequency range.

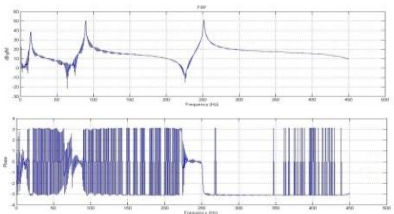


Fig. 19. FRF (MAGNITUDE AND PHASE)

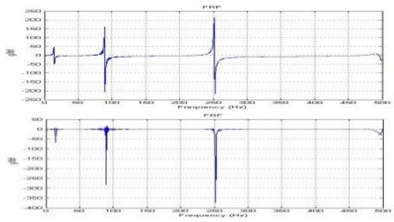


Fig. 20. FRF (REAL AND IMAGINARY)

The natural frequency can be identified by a phase shift of 1800 in the FFT phase (Fig. 21) [12, 13]. The advantage in using this method is that it allows you to monitor phase shifts and coherence. With this information, you can create operating deflection shapes to visualize the vibrating body.

The natural frequency can also be verified from the amplitude peaks in the CPS and APS graphs (Fig. 22 and Fig. 23 respectively).

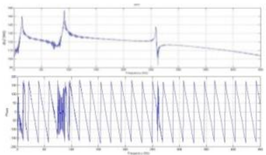


Fig. 21. FFT (MAGNITUDE AND PHASE)

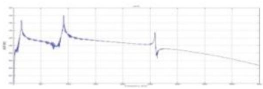


Fig. 22. CPS

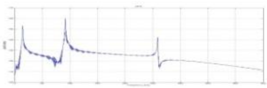


Fig. 23. APS

5.2. Shaker Test Results

The chirp test profile (Fig. 24 and 25) [2] shows the two peaks, which does not include the first mode as the shaker cannot detect low natural frequencies.

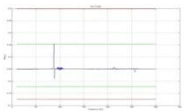


Fig. 24. SHAKER TEST PROFILE

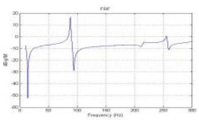


Fig. 25. FRF FOR SHAKER TEST

Table 2. NATURAL FREQUENCY FROM DIFFERENT METHODS

Mode	Natural Frequency (in Hz)					
	Euler's Method	MATLAB	ANSYS	Impact Test	Shaker Test	Acoustic
1	14.0985	16	14.9521	14.6484	-	18.75
2	88.3976	103	93.673	91.7967	87.73	93.75
3	247.4313	288	262.151	259.277	253.7	262.5
4	484.8412	566	513.323	509.277	-	515.625

5.3. Acoustic Test Results

Two set of readings are taken using the SLM (Fig. 26). The first is taken to measure the ambient conditions and the second to measure the excitation of the cantilever beam. The ambient reading gives us the data of the steady state excitations taking place in the room, which are eliminated from the data of the excited cantilever beam.

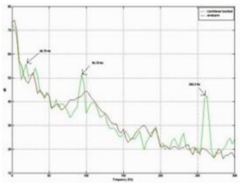


Fig. 26. FFT OF SLM READING

Shaker test is the most accurate of all the methods. Impact test although not as accurate as the shaker test, is a more convenient method of testing for smaller systems.

6. DISCUSSION OF RESULTS

6.1 Drive Point Measurement

- The phase loses 180 degrees [14] as we pass over the resonance and gains 180 degreesas we pass over the anti-resonance. (Fig. 27) [15]

Table 3.% ACCURAY OF DIFFERENT METHODS WITH RESPECT TO EULER'S METHOD

Mode	MATLAB	ANSYS	Impact Test	Shaker Test	Acoustic
1	86.5127	93.9455	96.0996	-	67.0071
2	83.4810	94.0322	96.1548	99.2448	93.9451
3	83.6041	94.0510	95.2125	97.4665	93.9099
4	83.2607	94.1255	94.9600	-	93.6507

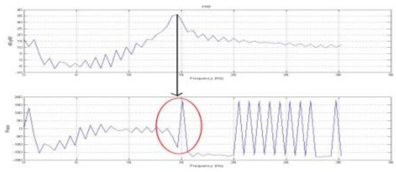


Fig. 27. PHASE CHANGE AT NATURAL FREQUENCY FOR DRIVE POINT MEASUREMENT

- All the peaks in the imaginary graph for the point 1 must point in the same direction (Fig. 28).

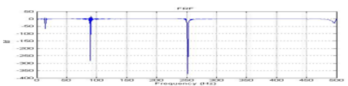
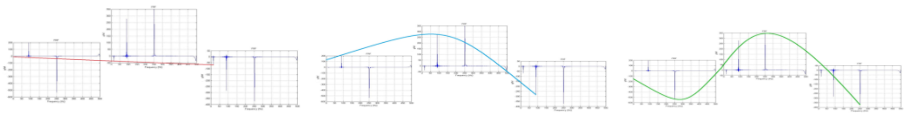


Fig. 28. IMAGINARY PART OF FRF FOR DRIVE POINT MEASUREMENT

6.2 Mode Shapes

The mode shapes of the cantilever beam can be plotted using the results from the three accelerometers. Joining the peak points of the first mode in the imaginary part of the FRF graph, for all three accelerometer readings, we achieve the first mode shape (Fig. 29 (a)).



(a) (b) (c)

Fig. 29. (a) FIRST MODE SHAPE, (b) SECOND MODE SHAPE (c) THIRD MODE SHAPE

Similarly, the mode shapes of other fundamental frequencies can also be found as shown in Fig. 29 (b) and (c).

6.3 Damping

Damping, a measure of energy dissipation in a vibrating system, has been recognized as playing a major role in the assessment of serviceability limit states. The method used here is the well-known, half-power bandwidth [10, 16]. The damping ratio is given by Eq. (5).

$$\xi = \frac{f_2 - f_1}{2f_{res}} \quad ...(5)$$

Table 4. DAMPING COEFFICIENT FOR THE FIRST THREE MODES AT THREE DIFFERENT POINTS

n	Damping Ratio		
	Point 1	Point 2	Point 3
1	0.0301	0.0304	0.03
2	0.0049	0.0048	0.0049
3	0.0018	0.00178	0.0024

Table 4 shows, the damping ratio is lesser for higher modes and there is little or no variation in the damping ratio values across the three points taken into consideration.

7. CONCLUSION

Structural vibration analysis is a multifaceted discipline that helps increase quality, reliability and cost efficiency in many industries. Analyzing and addressing structural vibration problems requires a basic understanding of the concepts of vibration, the basic theoretical models, time and frequency domain analysis, measurement techniques and instrumentation, vibration suppression techniques, and modal analysis. [17]

When the system under consideration is small, the best method of analysis is impact testing, as it is quick, convenient, the setup is simple and the hardware is cheaper in comparison. For larger systems, impact testing cannot be used because the energy given to the system by an impact can excite only a small range of frequencies. Using a shaker test different test profiles like chirp, dwell are used for a more thorough study of the system. Thus for larger systems shaker testing is used. On the other hand, for smaller systems the shaker test cannot be used as it may alter the dynamics of the system.

Acoustic test is more suitable when the system under consideration makes noise, on being excited, which is loud enough to be recorded by the SLM. Thus acoustic tests are used for condition monitoring.

8.ACKNOWLEDGMENTS

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