

Design of lobe pair profile of an external rotary lobe pump

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Abstract

Noncircular lobe rotor pair profile generation is complex and new form of external rotary lobe pump used in industry. Rotors used in lobe pumps are conjugate generated pair, generated from their respective pitch pairs. The pumping ratio of a lobe pump is also a function of pure geometry of the lobe and thus it is mainly governed by the pitch and deviation function. In present work, non-circularity of the pitch was considered as a main parameter. Direct Profile Design (DPD) method was used to develop identical noncircular pitch pair. The noncircular pitch pair was modified to get generated pair by applying envelope theory and deviation function method. Lobe pair with different noncircular pitch functions was obtained for a given deviation function. The zero interference of the generated conjugate lobe pair at different orientations was also verified using high end software. Specific flowrate formula in the form of pitch function and deviation function was used to compare performance of the lobe profiles developed using different noncircular pitch functions.

Keywords: Noncircular pitch, Envelope theory, Deviation function, Direct profile design.

1 Introduction

Lobe rotor pair is used as a crucial component in many mechanical systems like blowers, compressors and pumps. Positive displacement of fluid occurs as the two rotors are in conjugate motion as shown in the Fig. (1). The two rotors are attached with a pair of driving and driven gear for continuous motion; reason being one rotor can drive the other rotor only for half cycle. The lobe rotor pair which is identical to a gear pair is a conjugate kinematic pairs with continuous motion without oscillation. It maintains same velocity at each point of their contact. Conjugate pairs are of two types, pitch pair and generated pair. Pitch pairs are used to evolve their respective generated pairs.

In 1998, Tong have developed an algorithm for generating identical noncircular pitch pairs of any order known as DPD method [1]. In 1999, Deviation Function method was developed by Tong and Yang [2] for generated profile from the existing pitch curve using envelope theory. In 1999, Tong had also carried out the flow rate analysis of a lobe pump by taking pumping ratio of lobe pump as the performance index. Pitch non-circularity and lobe non-circularity are introduced as two dimensionless parameters along with their design significance by Tong [3]. Generalized specific flow rate formula in terms of pitch and deviation function was derived by Yang in 2002 [4]. In 2005, Tong and yang designed a lobe pair with deviation function based on the flowrate requirement for circular pitch [5]. Deviation function was also used by Yang Shyue-Cheng to modify an elastic conjugate element used in rotary gear pump to decrease stresses involved and increasing its bending

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strength [6]. Sarah Warren used deviation function method to design a rotary engine and an apex seal profile thereby proving the versatility of this method [7]. Pitch non-circularity and lobe non-circularity are the dimensionless parameters which governs lobe slenderness near tips of lobe and centre of lobe respectively. In 2002, Yang derived specific flowrate formula in terms of pitch and deviation function and compared the flowrate for different values of lobe non-circularity by taking a circular pitch [4]. Hence, change in flowrate with change in lobe non-circularity was concluded. In the present paper, noncircular pitch is considered and for a given deviation function, variation in flowrate with different noncircular pitch functions is observed.

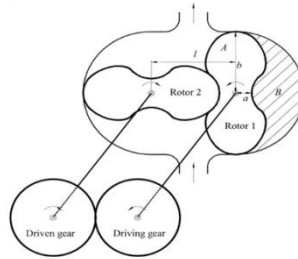


Figure 1:Lobe Pump

2 Direct-Profile-Design Method

A lobe rotor is designed in two steps in form of pitch pair design followed by generated pair. Correct pitch profile will cause uniform rolling action of the pitch pair. The conjugate pitch pair always makes a contact at a point where they share the same velocity. The DPD method was developed by S.H. Tong [1] for the generation of identical noncircular pitch pair.

Assume that rotor 1 and 2 as shown in Fig. (2) are two identical noncircular pitch pairs with a radius $r_1(\theta_1)$ and $r_2(\theta_2)$ respectively with monotonically increasing and C^1 continuity. Point A_1 is the starting point of the function $r_1(\theta_1)$. The profile in the second quadrant of pitch 1 is divided into two segments i.e. segment A_1E_1 and segment D_1E_1 .

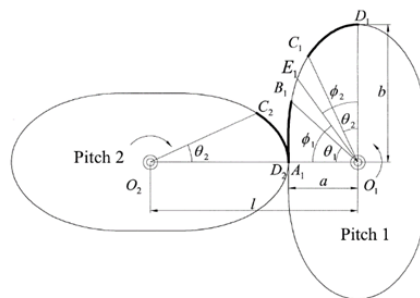


Figure 2: Noncircular Pitch Pair [1]

Segment D_1E_1 is same as segment D_2C_2 on rotor 2. Pitch pair is a kinematical, conjugate pair and thus profile segment A_1B_1 on rotor 1 should be conjugate with the profile segment D_2C_2 on rotor 2, i.e.

$$r_2\theta_1 = l - r_1(\theta_1) \quad (1)$$

θ_1 and θ_2 is the angular position of rotors on rotor 1 and 2 respectively. To ensure pure rolling condition, both the pitch should have the same velocity at the contact

point, i.e. $r_1 d\theta_1 = r_2 d\theta_2$. Thus, conjugacy and pure rolling are the two basic conditions need to be satisfied by all pitch functions. Combining the above two conditions relation between θ_1 and θ_2 can be given as

$$\theta_2 = \int \frac{r_1}{l - r_1} d\theta_1 \quad (2)$$

At point E1, where $r_1(\theta_1) = l/2$ (i.e. $r_1 = r_2 = l/2$) point B₁ and C₂ meets. At this point, θ_1 becomes ϕ_1 and $\phi_1 + \phi_2 = \pi/2$. Similarly profile segment in second quadrant of pitch 1 is composed of two segments as shown in Fig 2. These segments are given by,

$$r_1(\theta_1) = \text{segment A}_1\text{B}_1 = f(\theta_1), \quad \text{when } 0 \leq \theta_1 \leq \phi_1 \quad (3)$$

$$r_1(\theta_1) = \text{segment C}_1\text{D}_1 = \text{segment C}_2\text{D}_2, \quad \text{when } \phi_1 \leq \theta_1 \leq \pi/2 \quad (4)$$

3 Deviation Function Method

“Deviation Function (DF)” method was developed by Tong and Yang [2] for generating a conjugate profile from a given pitch profile. Generated lobe pair is not a pure rolling pair as pitch pair. Sliding occurs during the meshing of two lobes due to deviation between contact point of pitch pair P and contact point of generated pair G. In deviation function method, a function is selected as a pattern of deviation for different angular position. The generated lobe pair is absolute summation of pitch radius and deviation at respective angular position. Let p_1 and p_2 , represented by thin lines is a pair of original noncircular pitches; and g_1 and g_2 , represented by bold lines are corresponding generated profiles as shown in Fig. (3). The pitch pairs shown are identical and noncircular. At a given instance, the contact point of rotor’s pitch pair p_1 and p_2 is at point P. The locus of point P in moving coordinate frame O_{1xy} forms the pitch profile $r_1(\theta_1)$.

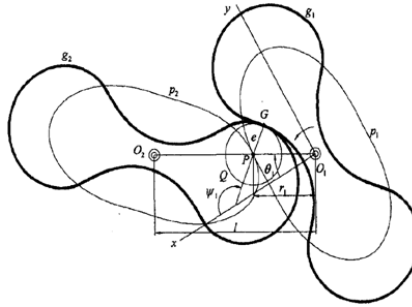


Figure 3: Pitch Pair and Generated Pair using Deviation Function method [2]

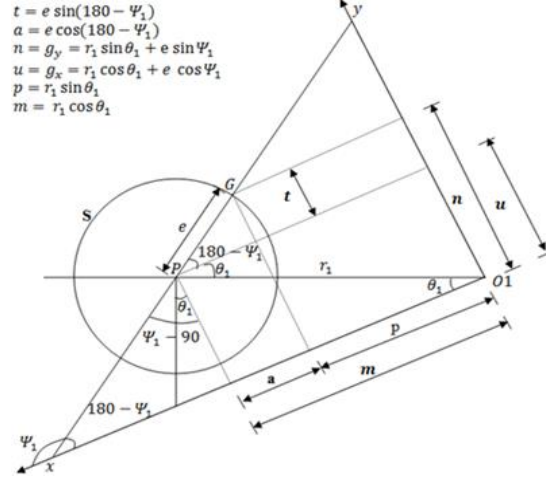
Similarly, let G be the contact point of the two generated profiles g_1 and g_2 corresponding to the same instant as that of pitch contact point P (here generated profiles g_1 and g_2 are considered to be as welded to pitch profiles p_1 and p_2 , respectively). The locus of point G in the moving coordinate frame O_{1xy} gives the generated profile. Thus, the deviation function method designs $G(\theta_1)$ by offsetting $P(\theta_1)$ by a distance equal to $e(\theta_1)$ as shown in Fig. (3). This offset distance is called deviation and its representation in form of angular position is called as deviation-function $e(\theta_1)$.

$$e(\theta_1) = P(\theta_1) - G(\theta_1) \quad \text{in } 0 \leq \theta_1 \leq \phi_1 \quad (5)$$

where, $e(\theta_1)$ is called the “deviation function”.

3.1 The Envelope Theory

The generated profile is the envelope of the family of the generating circles S on the O_{1xy} frame. The resultant envelope profile is the envelope generated by the circles with radius equal to e and centre as a point on pitch profile. The family of the generating circles S on the O_{1xy} frame is obtained by applying envelope theory as shown in Fig. (4). Point P is the contact point of two pitch curves and G is the contact point of generated curves (circles) S.



$$\begin{aligned} t &= e \sin(180 - \psi_1) \\ a &= e \cos(180 - \psi_1) \\ n &= g_y = r_1 \sin \theta_1 + e \sin \psi_1 \\ u &= g_x = r_1 \cos \theta_1 + e \cos \psi_1 \\ p &= r_1 \sin \theta_1 \\ m &= r_1 \cos \theta_1 \end{aligned}$$

Figure 4: Generating circle (S) with radius (e)

The family of curve is described by the equation in form $F(x, y, d) = 0$. According to the envelope theory of second type, F should be differentiated with respect to d . Eliminating d and reducing equation in form of x and y gives the equation of the envelope. Here, the family of curve is a family of generating circles S. The equation of family of circle is given by,

$$S(g_x, g_y, \theta_1) = (g_x - r_1 \cos \theta_1)^2 + (g_y - r_1 \sin \theta_1)^2 - e^2 = 0 \quad (6)$$

The envelope of generating circle is given by,

$$\frac{\partial S_1}{\partial \theta_1} = 0 \quad (7)$$

The desired envelope profile is obtained by solving the two equations, Eq. (6) and (7) simultaneously [7]. The alternate method to represent the family of curve is by taking each parameter in parametric form. Thus, describing parametric equations of same circle as in Eq. (6) and taking ψ_1 as parameter,

$$(g_x - r_1 \cos \theta_1) = e \cos \psi_1 \text{ and } (g_y - r_1 \sin \theta_1) = e \sin \psi_1$$

$$g_x = r_1 \cos \theta_1 + e \cos \psi_1 \quad (8)$$

$$g_y = r_1 \sin \theta_1 + e \sin \psi_1 \quad (9)$$

A point (g_x, g_y) is on the envelope curve only when,

$$\begin{vmatrix} \frac{\partial g_x}{\partial \psi_1} & \frac{\partial g_x}{\partial \theta_1} \\ \frac{\partial g_y}{\partial \psi_1} & \frac{\partial g_y}{\partial \theta_1} \end{vmatrix} = 0 \quad (10)$$

Simplifying,

$$r_1' + r_1 \tan(\psi_1 - \theta_1) = -e \sqrt{1 + \tan^2(\psi_1 - \theta_1)} \quad (11)$$

Squaring and rearranging terms, a quadratic equation in terms of $\tan(\psi_1 - \theta_1)$ is finally represented as,

$$\psi_1 = \theta_1 + \tan^{-1} \left(\frac{-r_1 r_1' \pm e' \sqrt{r_1^2 + r_1'^2 - e'^2}}{r_1^2 - e'^2} \right) \quad (12)$$

Deviation function $e(\theta_1)$ is provided in the range $0 \leq \theta_1 \leq \phi_1$ and the deviation function has to be zero at the conjunction point for ensuring C' continuity [2] (where $\theta_1 = \phi_1$), i.e.

$$e(\phi_1) = 0 \quad (13)$$

A deviation function $e(\theta_1)$ is satisfying Eq. (13) with many possibilities. Few possibilities satisfying the above conditions reported in [2] are as under:

$$e(\theta_1) = (\phi_1 - \theta_1) f_1(\theta_1) \quad (14a)$$

$$e(\theta_1) = \sin(\phi_1 - \theta_1) f_2(\theta_1) \quad (14b)$$

$$e(\theta_1) = (e^{(\phi_1 - \theta_1)} - 1) f_3(\theta_1) \quad (14c)$$

4 Concept of flowrate

Flowrate of pump is analysed using its dependent parameters. Let B denote the pocket area and A denotes the rotor area as per Fig. (1). The pocket area marked with hatched lines and it is an area between the housing and lobes where the fluid is normally trapped. Let q_i be the initial delivery volume per rotor revolution. If there is no leakage during the operation then q_i is expressed as,

$$q_i = 2(\pi b^2 - A)w \quad (15)$$

Where, b is maximum radial length, A is area and w = thickness of rotor.

The pump size V can be expressed as,

$$V = (\pi b^2 + 2bl)w \quad (16)$$

The pumping ratio is the ratio of delivery volume to total volume. Thus, pumping ratio r_p is obtained using Eq. (15) and (16),

$$r_p = \frac{q_i}{V} = \frac{2(\pi b^2 - A)}{(\pi b^2 + 2bl)} \quad (17)$$

It is observed that the pumping ratio is dependent on the flowrate and total volume which depends only on the geometry of rotor. Rotor's geometry is a function of both rotor's pitch and deviation functions [3]. Therefore, it is reasonable to suggest that following actions can be taken to improve pumping ratios:

- Changing the pitch function: Usually, the pitch curve is always a circle. Pitch curve with different noncircular functions responds in terms of geometrical changes and ultimately variation in flowrate.
- Changing the deviation function: Different deviation function satisfying Eq. (13) can be selected keeping the pitch function constant to observe variation in flowrate with change in deviation function.
- Different combinations of pitch function and deviation functions.

4.1 Specific Flowrate

The flowrate of a lobe pump is a function of angular position and varies periodically. During the meshing of the two lobes, the position of contact point change which leads to change in magnitude of flowrate. Pumping flowrate becomes a major

criterion to judge lobe pump performance after its design. Flowrate of a lobe pump can be defined as “delivery volume of pumping fluid per unit time”. In 2002 Yang derived the specific flowrate formula in terms of pitch and deviation function. This specific flowrate is a dimensionless parameter given by, $f = F/nV$ where n is real lobe frequency and V is pump size. The specific flowrate formula in terms of pitch and deviation function is given as

$$f = \frac{\pi l(b^2 - r_1(l - r_1) - e^2)}{(l - r_1)(\pi b^2 + 2bl)} \quad (18)$$

5 Flowrate analysis of Lobe Profile

Flowrate analysis of the lobe pump with circular as well as non-circular pitch function is carried out in this section. All the pitch profiles are generated using DPD method as discussed earlier for same centre to centre distance between lobes. The developed pitch profile was modified to generated profile using deviation function method. Daniel C.H. Yang et. al [4] reported the flowrate analysis of lobe profile generated using different deviation function method. They reported effect of lobe non-circularity and number of lobes on the specific flowrate. However, the analysed profiles were all made using a circular pitch. The effect on specific flowrate of the lobe profiles with change in pitch function is discussed in subsequent section using the Eq. (18).

5.1 Flowrate analysis of lobe pump with circular pitch

5.1.1 Generation of Circular Pitch Pair Profile

Presently, lobe rotor pair in lobe pumps with circular pitch profiles is commonly used in industries. A pitch pair with circular pitch having radius of lobe 1 and lobe 2 are as $r_1 = r_2 = 1$ as shown in Fig. (5). O_1O_2 is the common normal to pitch profile at point A for a particular velocity ratio.

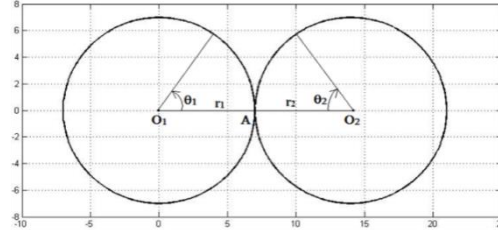


Figure 5: Circular Pitch Pair

5.1.2 Generation of lobe pair using deviation function method

Let the deviation function is an arbitrary pick $e(\theta_1) = (\phi_1 - \theta_1)(c_0 + c_1\theta_1)$ having $\phi = 45^\circ$ for $f_1(\theta_1) = c_0 + c_1\theta_1$ where c_0 and c_1 are coefficients which governs the geometry of rotor. The coefficient c_0 governs width of the rotor at its central portion. Let value of c_0 be $l/(9\phi_1)$ and $c_1 = l/(9\phi_1^2)$. Results shows that larger the value of c_0 more slender will be the rotor. The rotor with smaller c_1 is sharper at tips than the rotor with larger value of c_1 . The small circle on periphery of pitch circles are the generating circles as shown in Fig. (6). These circles are used to modify pitch circle to generated profile.

Inputs:

1. Pitch function, $r_1(\theta_1) = l/2$, where $a = b = 7$, $\phi_1 = 45^\circ$, $r_2(\theta_1) = l - r_1(\theta_1)$,

$$\theta_2 = \int \frac{r_1}{l - r_1} d\theta_1$$

2. Deviation function, $e(\theta_1) = (\phi_1 - \theta_1) \left(\frac{l}{9\phi_1} + \frac{l}{9\phi_1^2} \theta_1 \right)$, $0 \leq \theta_1 \leq \phi_1$

$$e(\theta_1) = \|P(\theta_1) - G(\theta_1)\|, \text{ for } 0 \leq \theta_1 \leq \phi_1$$

3. Value of r_1 and e in Eq. (18) for specific flowrate

Output: Generated profile and specific flowrate

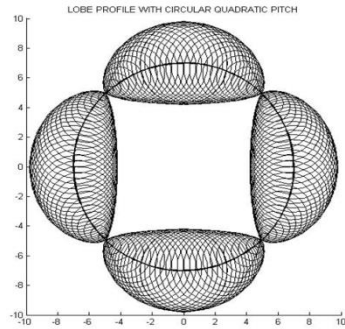


Figure 6(a): Generated profile using Circular Pitch function

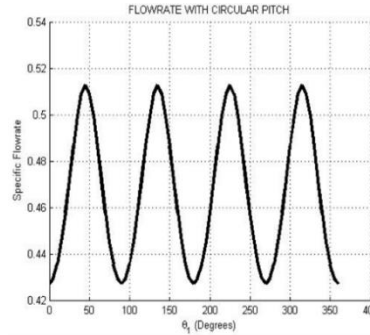


Figure 6(b): Flowrate of Generated profile in (a)

5.2 Flowrate analysis of lobe pump with linear pitch

5.2.1 Generation of Linear Pitch Pair Profile (DPD method)

Let value of r_1 varies from $4.6667 < r_1 < 9.3333$. Initial value of $r_1(\theta_1) = q_1(\theta_1)$ is a linear function $q_1(\theta_1) = a + \theta_1/b \ni q_1(0) = a$. Here, $l = a + b = 14$, also $q_1(\phi_1) = l/2 \Rightarrow \phi_1 = 21.7774^\circ$, by substituting $q_1(\theta_1)$, ϕ_1 and l in $\int_0^{\phi_1} \frac{l}{l - q} d\theta_1 = \alpha\pi/2 \Rightarrow \alpha = 23.921 \neq 1$. Thus $f_1(\theta_1) = q_1(\alpha\theta_1)$ where θ_1 is linearly spaced between $0 \leq \theta_1 \leq \phi_1/\alpha$ in n steps. This is the first segment. There exists θ_2 in the range $\pi/2 \leq \theta_2 \leq \phi_1/\alpha$ corresponding to these n values of θ_1 and the relation between θ_1 and θ_2 is given by $\theta_2 = \int \frac{r_1}{l - r_1} d\theta_1$. For second segment the function becomes $q_2(\theta_2) = l - q_1(\theta_1)$ within the range $\pi/2 \leq \theta_2 \leq \phi_1/\alpha$. These two segments are same as segments A_1E_1 and D_1E_1 in Fig. (2) which makes $1/4$ of the profile. By mirroring these segments, the complete profile of pitch 1 is obtained and pitch 2 is obtained by geometric transformation of pitch 1 as shown in Fig. (7)

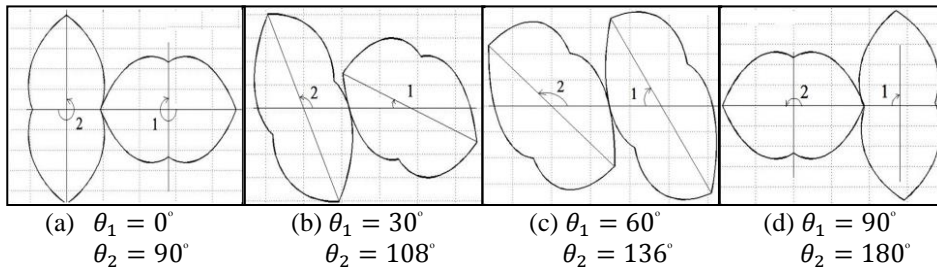


Figure 7: Noncircular pitch pair profile with linear function at 4 different angles

5.2.2 Generation of lobe pair using deviation function method

A noncircular pitch with linear function as obtained in Sec. 5.2.1 is modified to generated profile using the same deviation function used to modify the circular pitch. The pitch with linear function is generated using DPD method as shown in Fig. (8(a)).

Inputs:

1. Pitch function, $r_1(\theta_1) = a + \theta_1/b$, where $a = 4.667, b = 9.333, \phi_1 = 52.162^\circ$,
2. Deviation function, $e(\theta_1) = (\phi_1 - \theta_1) \left(\frac{l}{9\phi_1} + \frac{l}{9\phi_1^2} \theta_1 \right), 0 \leq \theta_1 \leq \phi_1$
3. Value of pitch and deviation function in Eq. (18)

Output: Generated lobe profile (Fig. (8(a))) and specific flowrate (Fig. (8(b))).

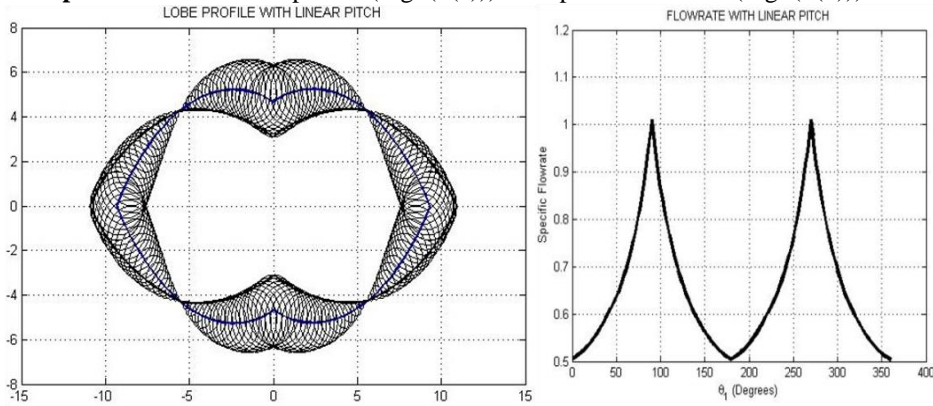


Figure 8(a): Generated profile using linear pitch function

Figure 8(b): Flowrate of generated profile

5.3 Flowrate analysis of lobe pump with quadratic pitch

5.3.1 Quadratic Pitch Pair Profile Generation

Let $a = 4, b = 10, l = 14$ and the initial design for the pitch function is $q_1(\theta_1) = r_1(\theta_1) = l/3 + 0.1834l\theta_1^2$ where, $\phi_1 = 54.6194$.

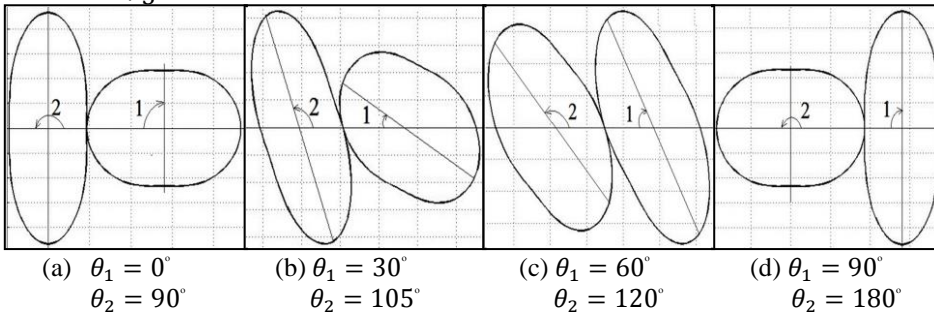


Figure 9: Noncircular pitch profile with quadratic pitch function at 4 different angles

5.3.2 Generation of lobe pair using deviation function method

The noncircular quadratic pitch profile used to generate the lobe profile is discussed in Section 5.3.1.

Inputs:

1. Pitch function, $r_1(\theta_1) = l/3 + 0.1834l\theta_1^2$, where $l = 14, \phi_1 = 54.6194^\circ$

2. Deviation function, $e(\theta_1) = (\phi_1 - \theta_1) \left(\frac{l}{9\phi_1} + \frac{l}{9\phi_1^2} \theta_1 \right)$, $0 \leq \theta_1 \leq \phi_1$.

3. Value of $r_1(\theta_1)$, $e(\theta_1)$ in specific flowrate equation

Output: Generated lobe profile (Fig. (10(a))) and specific flowrate (Fig. (10(b))).

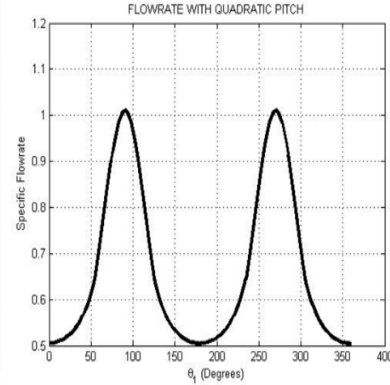
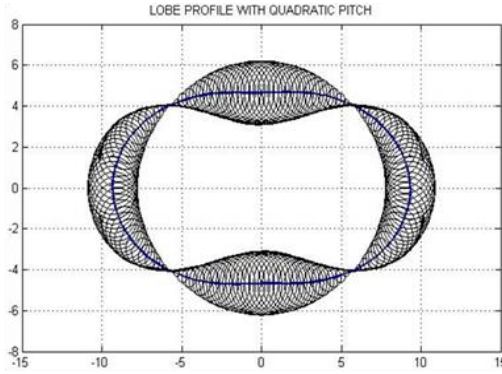


Figure 10(a): Generated profile using quadratic pitch function

Figure 10(b): Flowrate of generated profile

5.4 Flowrate analysis of lobe pump with Parabolic Pitch

5.4.1 Parabolic Pitch Pair Profile generation

Let $a = 4$ and $b = 10$ and the initial function of the pitch profile be a parabolic function, $q_1(\theta_1) = 3 + (\theta_1 + 1)^2$, $\phi_1 = 57.2958$.

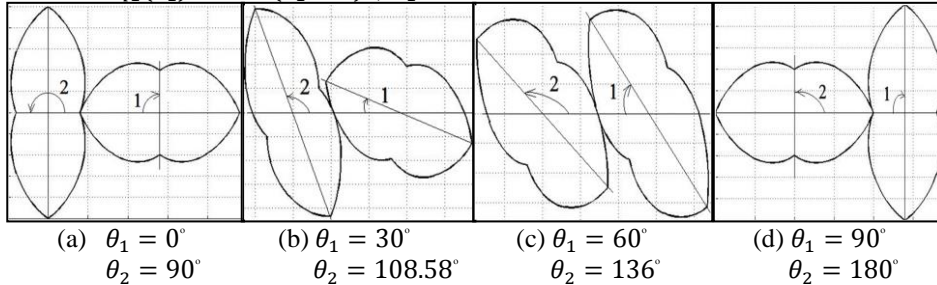


Figure 11: Noncircular pitch pair profile with parabolic function at different angles

5.4.2 Generation of lobe pair using deviation function method

The noncircular parabolic pitch profile generated above is used to generate the lobe profile.

Inputs:

1. Pitch function, $r_1(\theta_1) = 3 + (\theta_1 + 1)^2$, where $l = 14$, $\phi_1 = 57.2958$

2. Deviation function, $e(\theta_1) = (\phi_1 - \theta_1) \left(\frac{l}{9\phi_1} + \frac{l}{9\phi_1^2} \theta_1 \right)$, $0 \leq \theta_1 \leq \phi_1$.

$$e(\theta_1) = P(\theta_1) - G(\theta_1), \text{ for } 0 \leq \theta_1 \leq \phi_1$$

3. Value of $r_1(\theta_1)$ and $e(\theta_1)$ in Eq. (18)

Output: Shown in Fig. (12(a) & (b))

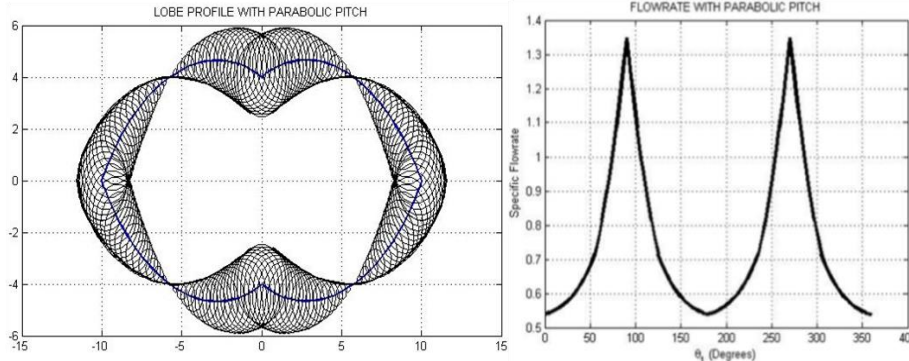


Figure 12(a): Generated profile using Quadratic Pitch function

Figure 12(b): Flowrate of the Generated profile in (a)

5.5 Conclusion

Pitch profiles other than circular pitch are designed using direct profile design method, generated lobe profile are developed using concept of deviation function method and envelope theory. The specific flowrate is plotted for each profile generated. It is apparent from Fig. 6(b), 8(b), 10(b) and 12(b) that the specific flowrate achieved for profiles generated using noncircular pitch is higher than the specific flowrate of the profile generated using circular pitch with the same deviation function. The specific flowrate for all four profiles at 20 equidistance angles between 0 to 90 is summarized in Table 1.

Sr. No.	Specific Flowrate of 1 st Quadrant (0°-90°)			
	Noncircular linear function	Quadratic function		Noncircular parabolic
		Circular	Noncircular	
1	0.5051	0.4276	0.5051	0.53939
2	0.51178	0.4297	0.50615	0.54451
3	0.52068	0.43584	0.50936	0.55193
4	0.53191	0.44555	0.51499	0.56207
5	0.54552	0.45805	0.52346	0.5754
6	0.56147	0.47224	0.53543	0.59249
7	0.57968	0.48672	0.55187	0.61402
8	0.59995	0.49977	0.57417	0.64082
9	0.622	0.50935	0.60436	0.67398
10	0.6454	0.51312	0.64541	0.71491
11	0.64541	0.51312	0.6454	0.71491
12	0.66985	0.50935	0.69528	0.76266
13	0.69595	0.49977	0.74832	0.81547
14	0.7246	0.48672	0.80271	0.87379
15	0.75676	0.47224	0.8561	0.938
16	0.79348	0.45805	0.90575	1.0084
17	0.83586	0.44555	0.94866	1.0849

18	0.88515	0.43584	0.9819	1.1675
19	0.94274	0.4297	1.003	1.2557
20	1.0102	0.4276	1.0102	1.3485

Note: The specific flowrate of circular pitch increases till ϕ_1 and then decrease back till it reaches 90° . While for noncircular pitch the flowrate keeps increasing till 90° .

Table 1: Specific Flowrate for different Profiles with centre distance 14.

The specific flowrate attained for a noncircular parabolic pitch is greater than that profile generated using noncircular linear or noncircular quadratic pitch functions. Thus, the effect of pitch function on specific flowrate of the generated profile was observed for same deviation function.

References

- [1] D.C.H. Yang, S.H. Tong, Deviation-Function Based Pitch Curve Modification for Conjugate Pair Design, ASME journal of Mechanical Design 121 (4), pp-579-586, 1999.
- [2] S. H. Tong, D.C.H. Yang, On the generation of new lobe pumps for higher pumping flow rate, Mechanism and Machine Theory, 35, pp- 997-1012, 2000.
- [3] S. H. Tong, D.C.H. Yang, On the generation of new lobe pumps for higher pumping flow rate, Mechanism and Machine Theory, 35, pp-997-1012, 2000.
- [4] Daniel C.H. Yang, Shih-Hsi Tong, "The specific flow rate of deviation function based lobe pumps—derivation and analysis", Mechanism and Machine Theory 37, pp-1025–1042, 2002.
- [5] Shih-Hsi Tong, Daniel C. H. Yang, "Rotor Profiles Synthesis for Lobe Pumps With Given Flow Rate Functions" ASME Journal of Mechanical Design 127, pp-287-294, 2005.
- [6] Yang Shyue-Cheng, "Applying envelope theory and deviation function to tooth profile design", Mechanism and Machine Theory, 42, pp-262–274, 2007.
- [7] John W. Ruther, "Geometry of Curves", Chapman & Hall/CRC, pp- 246-255.
- [8] Sarah Elizabeth Warren, "New Rotary Engine Designs by Deviation Function Method", Electronic Thesis and Dissertations, UCLA Ph.D., Mechanical Engineering 0330UCLA, 2012.