Design of a static balancing mechanism for coordinated motion of an external load

Aravind Baskar, Gurunathan Saravana Kumar, Sandipan Bandyopadhyay

Abstract

This paper presents the design of a mechanism that allows for the static balancing of an external load, moving in coordination with the actuator of the mechanism. The load is balanced with the help of linear springs mounted on the links of the mechanism. The springs are allowed to be of non-zero initial length, thus making the physical implementation of them easier, while requiring some analytical approximations in their modelling. The two requirements of the design, namely, kinematic coordination, and static balancing, are achieved via three-position closed-form synthesis, and numerical optimisation using a software tool, respectively. A complete case-study is presented as an illustration of the proposed method.

Keywords: Static balancing, kinematic synthesis, mechanism design.

1 Introduction

A “statically balanced” mechanism is one which needs no effort (in the absence of friction and other external forces) to move it, within a prescribed range. Typically, this is achieved by attaching springs to the links of the mechanism, in such a manner that when the mechanism moves within the prescribed range, the corresponding changes in the gravitational potential energy and the elastic potential energy negate each other. Such mechanisms have the advantage of requiring lesser external efforts in moving them. The required effort is also more uniform, as the nonlinear variations in the gravitational load on the input link due to the motion of the links are reduced, if not completely eliminated.

There has been several studies aiming at the development of statically balanced planar mechanisms, such as the four-bar [1-2]. In these cases, the objective of balancing is confined to the mechanism itself. In contrast, some other mechanisms have been developed with the purpose of balancing an additional external load, in the form of a "payload" of constant mass [3]. A multi-stage scissor linkage has been

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utilised to achieve the vertical motion of the given payload. Also, both the tension and compression springs have been used, with zero effective initial length.

The present work is related to [3] in the sense that it also guides a given payload of fixed mass over a given range of vertical motion. However, it includes a series of novel features: the vertical guidance of the payload is achieved by a suitably designed four-bar mechanism, which is much simpler in its design and construction; the static balancing of the payload is also achieved by the same four-bar, augmented with two appropriately placed linear springs, which are both in tension, and have non-zero initial lengths. As detailed later in the paper, these restrictions imposed on the design require certain analytic approximations to be included in the formulation of the design problem. These approximations, however, are justified quantitatively, as they introduce negligible error in the analysis - a drawback that is significantly superseded by the simplicity and ease that they bring into the practical implementation of the mechanism.

The rest of the paper is organised as follows: Section 2 presents the kinematic synthesis of the mechanism to support and guide payload; Section 3 describes the formulation based on the potential energy for static balancing of the synthesised mechanism; Section 4 presents the solution scheme using a numerical optimization technique and analysis of error; and Section 5 concludes the paper.

2 Kinematic synthesis for the vertical guidance of a payload

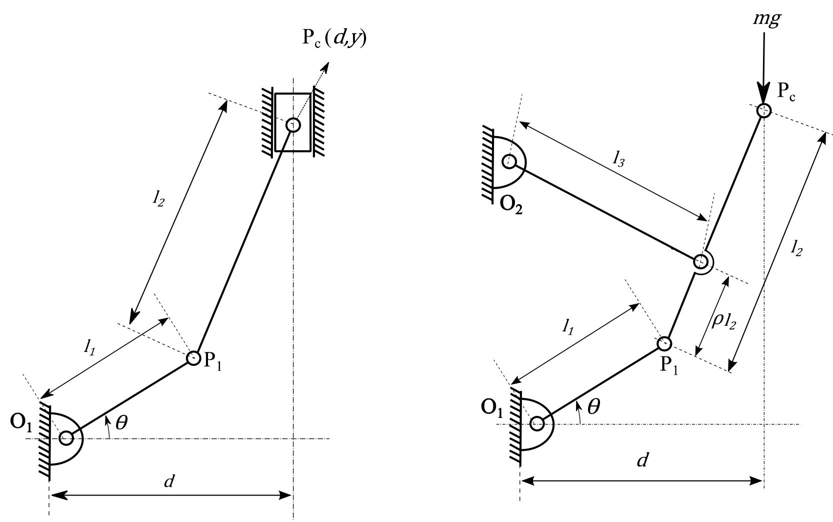
The desired objective for the kinematic synthesis step is to design a mechanism that guides a payload along a vertical straight line, as the crank turns through a given angle. It is easy to achieve the same using a slider-crank mechanism. However, from the perspective of reducing friction and the associated wear at the prismatic joint, it is desirable to have a mechanism with only rotary joints, while meeting the given motion objective. This is achieved in a two-stage process, as described below. The procedure is adopted from [4]. First, a slider crank mechanism is designed and then a four-bar mechanism generating approximately the same motion as that of the slider-crank mechanism is synthesised.

A schematic of the slider-crank mechanism is shown in Fig. 1a. It is desired that as θ changes from θ_i to θ_f, the payload at $\mathrm{P_c}(x, y)$ moves on a straight line $x = d$, while $y \in [y_i, y_f]$. A three-position synthesis procedure is adopted, using the two terminal points as well as the mid-point of the interval as the set of precision points. The associated numerical details are given in Table 1. By analogy with the Freudenstein's equation for the four-bar mechanism, a synthesis equation is derived as follows:

$$K_1 + K_2 \cos\theta + K_3 y \sin\theta = y^2 \quad (1)$$

where $K_1 = l_2^2 - l_1^2 - d^2$, $K_2 = 2l_1 d$ and $K_3 = 2l_1$. Substituting the three accuracy points (θ_i, y_i), $i = 1, 2, 3$ and solving the resulting three algebraic equations in

K_1, K_2 and K_3 simultaneously, the link lengths of the slider-crank are obtained, as shown in Table 2.



a) Slider crank mechanism b) Equivalent four-bar mechanism

Figure 1: Synthesis of an approximate straight-line motion generating linkage.

Table 1: Objective for the kinematic synthesis of the slider crank mechanism

S.No.	*Precision point*	*Crank angle, θ (radians)*	*Vertical displacement, y (m)*
1	Start point	$-\pi/6$	0.2
2	Middle point	0	0.3
3	End point	$\pi/6$	0.4

Table 2: Design of the four-bar mechanism

S.No.	*Parameter*	*Value (m)*
1	l_1	0.200
2	l_2	0.300
3	d	0.187
4	l_3	0.276
5	O_1	(0, 0)
6	O_2	(-0.080, 0.090)
7	ρ	0.300

A four-bar mechanism generating approximately the same motion of P_c as the slider-crank mechanism is then synthesised following [4]. It involves essentially determining the coupler length l_3 and the corresponding pivot, O_2. In the process, one has to determine the variable ρ (see Fig. 1b). A high value of ρ leads to large link lengths, while small values of ρ brings the joints closer, making the fabrication of the mechanism difficult, and hampers the transmission. As a reasonable compromise between the two aspects, ρ is chosen to be 0.3, thereby completing the kinematic synthesis process. The generated coupler motion is compared with the ideal curve in the Fig. 2. It can be seen that within the range of motion $y \in [y_1, y_3]$, the design requirement is satisfied with a maximum error of 0.0005m in the x-coordinate within the region of interest.

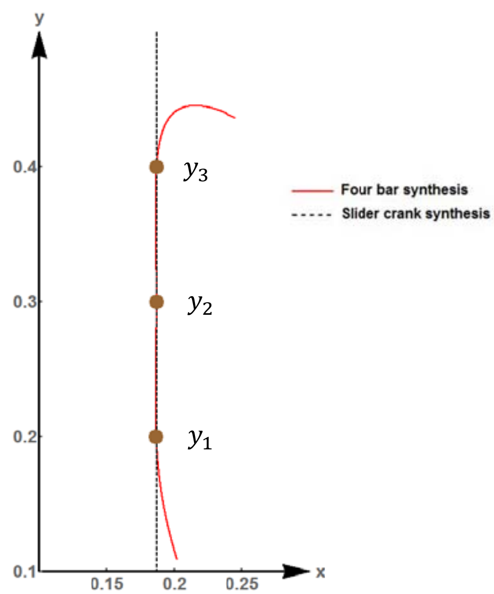


Figure 2: Comparison of the generated path against the desired one.

3 Static balancing of the linkage

Static balancing works on the principle, that if the total potential energy of the system remains constant in every configuration in the desired range of motion, no external work needs to be done to operate the system[1]. It is typically achieved by attaching spring(s) at strategic location(s) in the mechanism, such that the change in the gravitational potential energy of the system is compensated by an appropriate change in the elastic potential of the said spring(s). In the following, the same idea is used to balance an external gravitational force due to the payload applied at the point P_c, which is modelled via the corresponding potential energy. Two aspects of the design, however, can be considered novel:

(a) The springs used in this work are of finite initial length, making the transition from mathematical model to physical prototype much easier.

[1]A state of static equilibrium is assumed, alongside the usual assumptions of negligible friction, etc.

(b) The springs are used only in the tensile mode.

Typically, researchers use springs of zero initial-lengths (see, e.g., [2, 3]) which make the analysis simpler and exact, while rendering the fabrication somewhat complicated.

3.1 Derivation of the potential energy function corresponding to the payload

In this case, the payload is a particle of mass m attached to the point P_c. The potential energy V_{load} is derived as a function of the crank angle θ from basic principles $V = mgh$, where m is the mass of the payload in kg, g is the acceleration due to gravity in m/s^2 and h is the height in metre of the centre of mass of the load from the reference plane expressed as a function of θ.

$$V_{load} = mgh \tag{2}$$

Using Taylor's approximation about the mid-point $\theta = 0$ upto the fourth order, V_{load} is expressed as a polynomial in θ:

$$V_{load} \cong a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + a_4\theta^4 \tag{3}$$

This approximation helps the subsequent mathematical operations, while deviating from the actual value by 0.11% at the most within the desired range of motion (see Fig. 3). The constants a_0, a_1, a_2, a_3 and a_4 are functions of the link lengths and other design parameters, such as the fixed pivot co-ordinates.

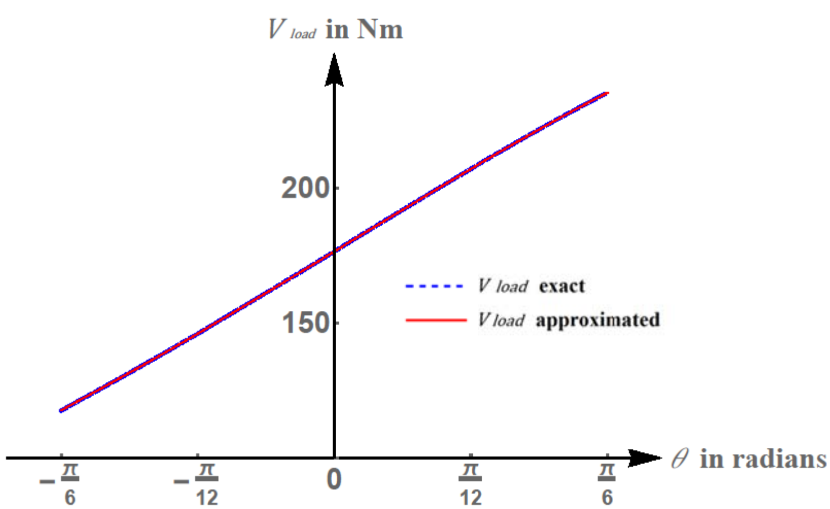


Figure 3: The external potential V_{load} and its approximation as in Eq. (3).

3.2 Derivation of the potential function of the springs

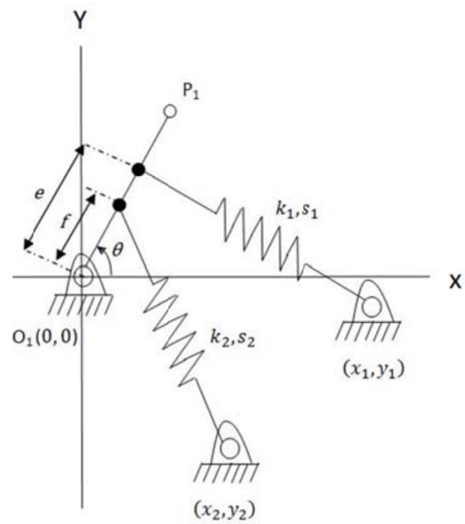


Figure 4: Linear coil springs attached to link 1 for balancing.

Fig. 4 is a schematic representation of the arrangement of the springs that achieves the static balancing of the payload. Let k_1 and $k_2$be the spring constants of the springs; and s_1 and s_2 be their free lengths, respectively. One end of the i^{th}spring is pivoted to the ground at (x_i, y_i), and the other end is pivoted to the crank, at a distance e (for $i = 1$) and f (for $i = 2$), respectively, from the point O_1. The potential energy of the springs can be expressed as functions of θ as follows:

$$V_{sp1} = \frac{1}{2}k_1(-s_1 + \sqrt{(x_1 - ecos\theta)^2 + (y_1 - esin\theta)^2})^2 \tag{4}$$

$$V_{sp2} = \frac{1}{2}k_2(-s_2 + \sqrt{(x_2 - ecos\theta)^2 + (y_2 - esin\theta)^2})^2 \tag{5}$$

Upon Taylor expansion about the mid-point $\theta = 0$ upto the fourth order, one obtains:

$$\begin{aligned} V_{sp1} \cong \frac{1}{2}k_1\Bigg(&2e\theta y_1\left(-1 + \frac{s_1}{\sqrt{e^2-2ex_1+x_1^2+y_1^2}}\right) + \left(s_1 - \sqrt{e^2 - 2ex_1 + x_1^2 + y_1^2}\right)^2 + \\ &\frac{e\theta^2(x_1(e^2-2ex_1+x_1^2+y_1^2)^{3/2}-s_1(-2ex_1^2+x_1^3-ey_1^2+x_1(e^2+y_1^2)))}{(e^2-2ex_1+x_1^2+y_1^2)^{3/2}} + \\ &\frac{1}{3}e\theta^3 y_1\Bigg(-\frac{3e(-2ex_1^2+x_1^3-ey_1^2+x_1(e^2+y_1^2))}{(e^2-2ex_1+x_1^2+y_1^2)^2} + \\ &\frac{(e^4-ex_1^3+x_1^4-e^2y_1^2+2x_1^2y_1^2+y_1^4-ex_1(e^2+y_1^2))(-s_1+\sqrt{e^2-2ex_1+x_1^2+y_1^2})}{(e^2-2ex_1+x_1^2+y_1^2)^{5/2}}\Bigg) - \\ &\frac{1}{12(e^2-2ex_1+x_1^2+y_1^2)^{7/2}}e\theta^4\Big(x_1(e^2 - 2ex_1 + x_1^2 + y_1^2)^{7/2} - s_1(-3ex_1^6 + x_1^7 - 4e^5y_1^2 + \\ &7e^3y_1^4 - 4ey_1^6 + 3x_1^5(e^2 + y_1^2) - 2x_1^4(e^3 + 5ey_1^2) + x_1^3(3e^4 + 4e^2y_1^2 + 3y_1^4) + \\ &x_1^2(-3e^5 + 6e^3y_1^2 - 11ey_1^4) + x_1(e^6 + e^4y_1^2 + e^2y_1^4 + y_1^6))\Big)\Bigg) \end{aligned} \tag{6}$$

Similar polynomial expression of degree 4 is obtained for V_{sp2}. These approximations make the analysis of non-zero initial length spring relatively easier.

3.3 Derivation of the root-mean square error of residual torque

From Eq. (3) and Eq. (6), the total potential energy of the system is given by[2]

$$V_{net} = V_{load} + V_{sp1} + V_{sp2} \tag{7}$$

The net torque G_{net} to operate the system within the range can be obtained by partial differentiation of V_{net} with respect to the input angle θ.

$$G_{net} = \frac{\partial V_{net}}{\partial \theta} \tag{8}$$

The system will be statically balanced over the range, when G_{net} is zero for all configurations over the range.

4 Numerical optimization

In this section, numerical optimization of $G_{net}$is carried out over the operating range. The details of the optimization are explained in the sub-sections.

4.1 Objective function

Root mean square value of the function G_{net} is considered to be the objective function for the optimization. The function G_{net} is a polynomial of degree 3 in θ; hence, the root-mean square value E_{rms} of the function can be obtained in the exact form via integration by θ over the range

$$E_{rms} = \sqrt{\frac{\int_{\theta_i}^{\theta_f} G_{net}^2 d\theta}{(\theta_f - \theta_i)}} \tag{9}$$

Integration is enabled by the Taylor's approximation of potential energy functions as described in Section 3.

4.2 Constraints

The following constraints arise from various practical considerations:

1. With regards to the springs, it is always preferable to have tensile springs and the maximum stiffness of the springs were limited to $10000 N/m$, i.e. $0 < k_1, k_2 \leq 10000 N/m$.

[2]The potential energy of the links was disregarded in comparison with that associated with the payload—an approximation which has been validated *post-facto*

2. Free length of the springs should always be greater than zero and were required to be greater than 0.1m for practical considerations i.e. $0.1m \leq s_1, s_2$.
3. Let the extended lengths of the tensile springs be s_1' and s_2'. In order to avoid the non-linear operational regime of the spring, care must be taken to ensure the extended length always lies above 120% of the free length. It is also desirable to have the extended length to be less than twice that of the free length for feasible operation, i.e. $1.2s_i \leq s_i' \leq 2s_i, i = 1, 2$.
4. The ground positions of the springs were restricted by the overall packaging constraints.
5. The pivoting points were forced to lie within 20-80% of the length along the crank i.e. $0.2l_1 \leq e, f \leq 0.8l_1$.

4.3 Design vector

The design vector $\boldsymbol{d}$ for the optimization includes the spring design parameters and spring position parameters as given in Eq. (10).

$$\boldsymbol{d} = \{k_1, k_2, s_1, s_2, x_1, y_1, x_2, y_2, e, f\} \tag{10}$$

4.4 Results

The optimisation problem defined above is solved for the payload m = 60 kg, using the built-in numerical optimiser routine NMinimize in the software Mathematica 10.0®, using the default setting of its internal parameters. The following results for the design parameters were obtained:

$k_1 = 10000\ N/m$, $k_2 = 5939.270\ N/m$, $s_1 = 0.181\ m, s_2 = 0.211\ m, x_1 = 0.200\ m, y_1 = 0.300\ m, x_2 = -0.200\ m, y_2 = 0.076\ m, e = 0.065\ m, f = 0.16\ m.$

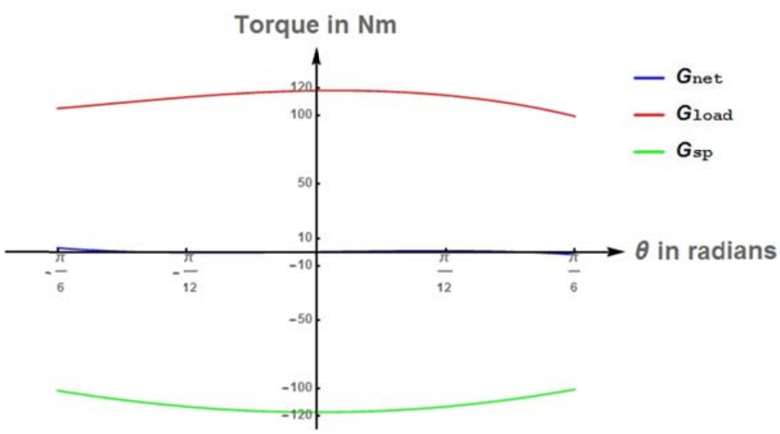


Figure 5: Comparison of the torque before and after balancing.

Maximum value of the torque due to the payload G_{load}, reflected on the crank before balancing is $117.821\ \mathrm{N/m}$. The corresponding value after balancing is $2.816\ \mathrm{N/m}$(see Fig. 5). Compensatory torque provided by the springs G_{sp} brought down the peak torque by 97.6%. The residual torque, which is less than 3%, may be expected to be provided by the friction in the system, thereby holding the system in place by itself, within the operating range.

5 Conclusions

This paper proposes a new design scheme for mechanisms that satisfy two criteria simultaneously, namely, the kinematic objective of guiding a given payload (in the form of a particle of constant mass) on a vertical path in coordination with the motion of the input link, and the static objective of balancing the corresponding gravity load. While the first objective is achieved using an established method, the second one employs certain novel measures, such as the use of two linear coil-springs of non-zero initial length, only in the tensile mode. These elements of the design make the fabrication of the physical mechanism easier. Certain analytic approximations have been employed to make the mathematical model of the mechanism amenable to analysis while accommodating the real-life engineering requirements and it has been ascertained quantitatively that the errors induced due to these are within tolerable limits. Efforts are underway to improve the mathematical model/solution techniques, leading to better results.

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