Finite element analysis and design of an asymptotically correct patch-actuator model

Shreya Banerjee, Sitikantha Roy

Abstract

The main aim of the present study is to develop a mathematical model of a patch actuator using variational asymptotic method. Patch actuators are based on inverse piezoelectric effect where an electrical voltage is used as an input to induce internal stresses or mechanical deformations.Variational asymptotic method is a mathematical approach which can be applied to any problem defined by an energy functional having one or more small parameters. In this work for analyzing a patch actuator model we have taken variation of electromechanical enthalpy functional. Starting with the three dimensional electromechanical enthalpy functional which is then approximated asymptotically using the slenderness of the structure as the small parameter to find out an equivalent one dimensional electromechanical enthalpy functional. The three dimensional components of the field variables has been recovered to get a good insight of both electrical and mechanical variables of the patch actuator model. There are wide range of applications of a patch actuator, starting from cantilever tip of an Atomic force microscopy to miniature robotics, damage detection, excitation and control of beam, energy harvesters to aircraft and aviation industry. We have validated our results with the simulation results obtained from ABAQUS and shows a very good agreement.

Keywords: Variational asymptotic method Patch actuator Piezoelectrics.

1 Introduction

Presently adaptive structures are receiving extensive attention in modern research. Structures which mainly integrate with actuators and sensors are taking the major role in this field. Piezoelectric patch actuators are based on inverse piezoelectric effect where an electric field as an input can induce internal stresses. Patch actuators have wide range of applications starting from miniature robotics, damage detection, excitation and control of beam, energy harvesters to aircraft and aviation industry. There are many theories discussed in the past to analyze an actuator model. Similarly, this paper discusses a new theory variational asymptotic method (VAM) to analyze a patch- actuator model. Variational asymptotic method was first proposed in 1979 by Berdichevsky[1]. Later, many applied the theory to model and analyze passive as well as active beams models[2,3]. Analyzing a beam like slender structure with a piezoelectric patch using a entirely different theory named variational asymptotic method will be the main contribution of the present paper. Variational asymptotic method has certain advantages over other methods as per discussed in the past

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Shreya Banerjee (Corresponding author)
Applied Mechanics Department/Research Scholar, Indian Institute of Technology-Delhi/Hauz Khaz, New Delhi-110016, E-mail:shreyabanerjee002@gmail.com.

Sitikantha Roy
Applied Mechanics Department/Assistant Professor, Indian Institute of Technology-Delhi/Hauz Khaz, New Delhi-110016, E-mail:sitiroy@gmail.com.

literatures. It is basically a combination of systematic and numerically easier method without any previous kinematic assumptions. The methodology splits the three dimensional actuator beam model into two dimensional cross sectional analysis and one dimensional beam analysis. Unlike the asymptotic method which are based on asymptotic expansions of the three dimensional quantities, variational asymptotic method applies the asymptotic expansion to the energy functional instead to the system of differential equations. Thus saving lot of computational time by eliminationg the small terms at the very functional level. There are lot of beam theories discussed in the past literature, but most of them are based on some previously mentioned kinematic assumptions. Some of them assume some distribution across the cross section for the 3D quantities in terms of the 1D quantities, the other methods are based on asymptotic expansions of the 3D qunatities while other category of models are based on Saint-Venant principle. The advantage of the present theory lies in effectively capturing the electromechanical coupling effect of any smart structure. To analyse the patch actuator model we have considered the model as a slender beam model. Geometrically slender structure mean where the ratio of the characteristic size of the cross section to the characteristic wavelength of the axial deformation is much much less than unity.

We have developed a mathematical model of a patch piezoelectric actuator with cantilever boundary conditions. The piezoelectric patch layer has been considered half the length of the total length of the structure. To capture the structural effects we have recovered the field variables at the mid cross section of the patch. To validate our results we have done a full scale 3D finite element analysis in ABAQUS and through post processing we have obtained the mechanical as well as the electrical variables. The result obtained from 3D finite element simulation done in ABAQUS has been compared with the result obtained from VAM and shows a very satisfactory match. The 2D cross sectional analysis combined with 1D beam analysis and a full scale 3D recovery using VAM can give highly efficient and comparable 3D results at lower computational cost. Thus this methodology proves to be well capable of giving a clearer and better insight about the resposes of the electro-mechanical variables for any realistic patch actuator model applied in various domain such as robotics, aerospace application, damage detection, energy harvestors etc.

2 Mathematical formulation

The patch actuator has been modelled as a slender beam model. The behaviour of a slender beam can be analysed by Hamilton’s principle.

$$\int_{t_1}^{t_2} [\delta(K-U) + \overline{\delta W}]dt = 0 \tag{1}$$

where K is the kinetic energy, U is the electric enthalpy for a piezoelectric material, W is the virtual work done by the applied loads or electric charges. The bar used above the W indicates that it need not be a variation of the functional.

The position vector of any point in an undeformed structure can be defined as,

$$\hat{\mathbf{r}}(x_1, x_2, x_3) = \mathbf{r}(x_1) + x_\alpha \mathbf{b}_\alpha \tag{2}$$

Similarly the position vector of any point for an deformed structure can be defined as,

$$\hat{\mathbf{R}}(x_1,x_2,x_3)=\mathbf{R}(x_1)+x_\alpha\mathbf{B}_\alpha(x_1)+w_i(x_1,x_2,x_3)\mathbf{B}_i(x_1) \tag{3}$$

The generalized 1D strains can be written as [4,5],

$$\gamma_{11}\mathbf{b}_1=\mathbf{b}_i\mathbf{B}_i.\mathbf{R}^{'}-\mathbf{r}^{'}$$

$$\kappa_i\mathbf{b}_i=\mathbf{b}_i\mathbf{B}_i.\mathbf{K}\text{ - }\mathbf{k} \tag{4}$$

For small local rotation we can express the strain as,

$$\Gamma_{ij}=1/2(F_{ij}+F_{ji})-\delta_{ij} \tag{5}$$

However for complete description of a patch actuator we also need electric field which is characterized by electric potential,

$$\mathbf{E}=-\nabla\varphi=-\frac{\partial\varphi}{\partial x_i}\mathbf{g}^i \tag{6}$$

Thus from Eqs. (5) and (6) we can get the expression for 3D strains as,

$$\mathit{\Gamma}=\mathit{\Gamma}_h\hat{w}+\mathit{\Gamma}_\in\in \tag{7}$$

where Γ_h, $\Gamma_\in$ are the operator matrices as given in Roy.et al[3].

The kinetic energy of the patch actuator model can be discretised as,

$$K=\frac{1}{2}\int_V\rho v^T v dV=K_{1D}+K^* \tag{8}$$

K_{1D} are the zeroth order kinetic energy terms and K^* are the higher order energy terms.

Similarly the virtual work due to the applied load can also be discretised as,

$$\delta\overline{W}=\delta\overline{W}_{1D}+\delta\overline{W}^* \tag{9}$$

We can thus write the Hamilton's principle as,

$$\int_{t_1}^{t_2}[\delta(K_{1D}+K^*\text{ - }U)+\delta\overline{W}_{1D}+\delta\overline{W}^*]dt=0 \tag{10}$$

Thus the 3D electromechanically coupled problem is discretized in terms of 1D displacements and rotations and 3D warping functions. The 3D warping functions are solved by asymptotic analysis of the variational statement.

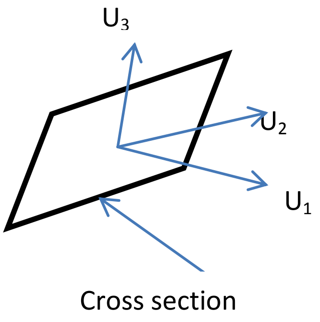

Figure 1: Schematic of the rectangular cross section of the patch actuator

2.1 Asymptotic analysis

Considering the zeroth order terms and neglecting the higher order terms we obtain,

$$\int_{t_1}^{t_2}[\delta(K_{1D}-\int_{0}^{L}U_{0}dx_{1})+\delta\overline{W}_{1D}]dt=0 \tag{11}$$

where U_0 is the zeroth order internal energy per unit span. The 3D warping function w_i and ϕ appears only in the U_0 and hence can be solved by the variational statement.

$$\delta U_0 = 0 \tag{12}$$

Thus solving we obtain the 1D constitutive model for the classical beam analysis of an actuator model,

$$\begin{bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = S \in - f^{a} = \begin{bmatrix} s_{11}s_{12}s_{13}s_{14} \\ s_{12}s_{22}s_{23}s_{24} \\ s_{13}s_{23}s_{33}s_{34} \\ s_{14}s_{24}s_{34}s_{44} \end{bmatrix} \begin{bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix} - \begin{bmatrix} f_1^{a} \\ m_1^{a} \\ m_2^{a} \\ m_3^{a} \end{bmatrix} \tag{13}$$

3 Results

The above developed theory for smart slender beam has been implemented into a 2D cross sectional analysis computer program VABS[3]. VABS is capable of generating equivalent one dimensional cross sectional stiffness constants. Those stiffness constant has been used as an input to the 1D beam analysis. We have modelled the patch actuator as an Euler-Bernoulli slender beam model. As an example problem we have considered a patch actuator model with 0.2m length of the aluminium layer with a PZT4 patch layer half of its length. The polarization is along the thickness directionTo verify our mathematical model we have compared our model with the 3D finite element model obtained from ABAQUS. In ABAQUS we have used 3D quadratic brick element to model the passive layer i.e the aluminium layer and quadratic piezoelectric element to model the PZT4 layer. We have given a voltage of 100V to the PZT4 patch layer. We have used tie constraint to assume perfect bonding between the two layers. The mechanical displacements, strains obtained from our model are compared with the 3D finite element model obtained from ABAQUS and shows a appreciable match. The patch actuator is entitled to give mechanical deformation for any amount of electrical input. Figure. 1, Figure. 2 and Figure. 3 shows the displacement comparison between VABS and ABAQUS of the mid cross section of the patch layer in x_1, x_2 and x_3 directions. Displacement in the transverse direction is more as compared to the other two directions. Figure. 4 shows the input voltage comparison which is zero for the passive layer and varying linearly in the active layer i.e in the patch layer. Figure. 5, Figure. 6, Figure. 7 shows the strain comparison in the three directions and Figure. 8 shows the stress comparison in the x_1 direction.

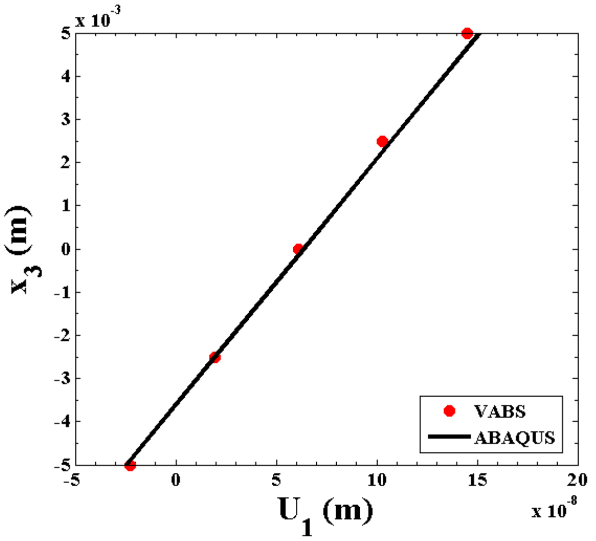

Figure 2: Comparison between the displacement of the cross section in x_1 direction.

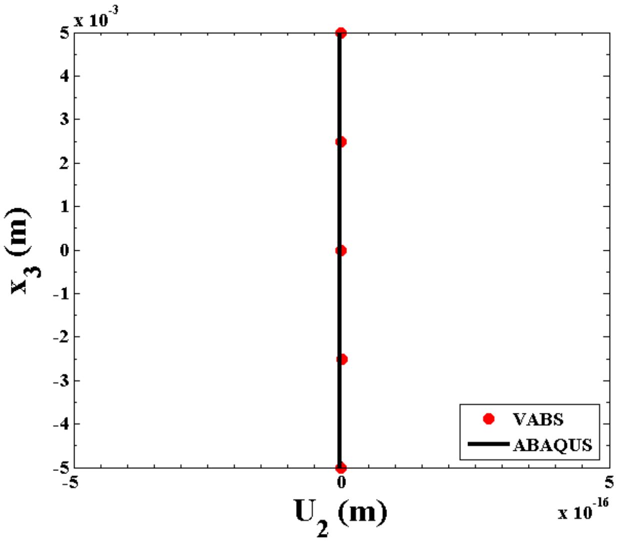

Figure 3: Comparison between the displacement of the cross section in x_2 direction.

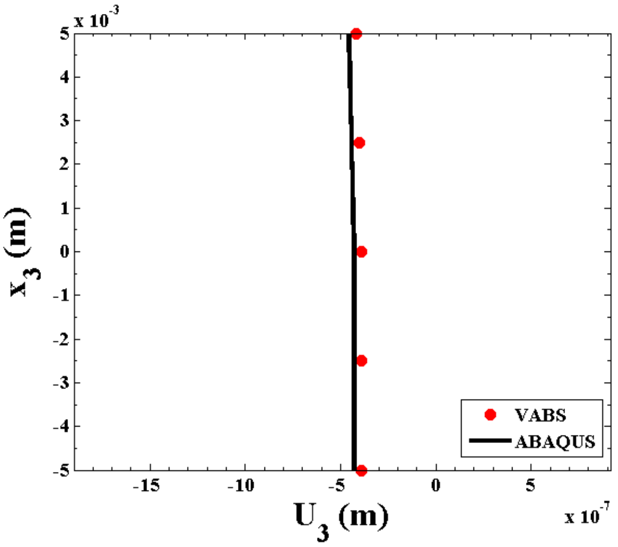

Figure 4: Comparison between the displacement of the cross section in x_3 direction.

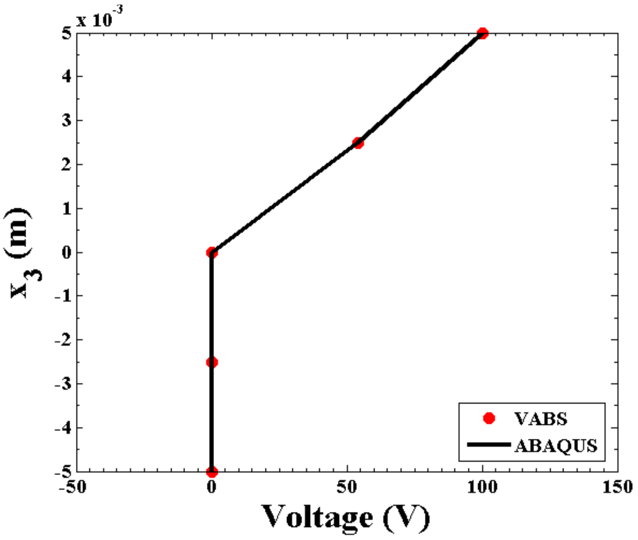

Figure 5: Comparison between the voltage distribution across the cross section.

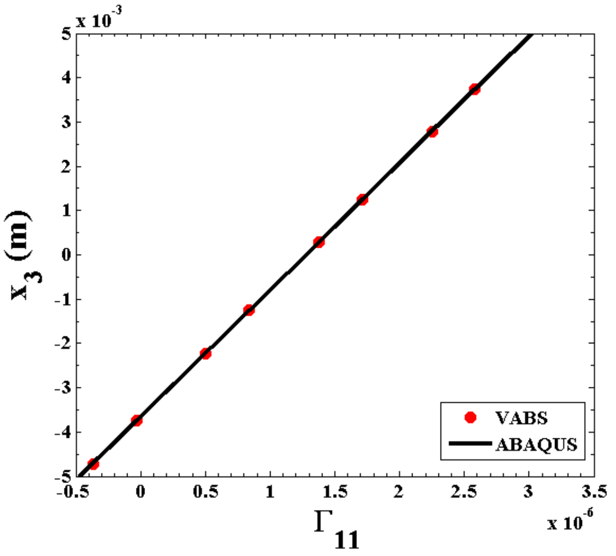

Figure 6: Strain distribution in x_1 direction along the thickness

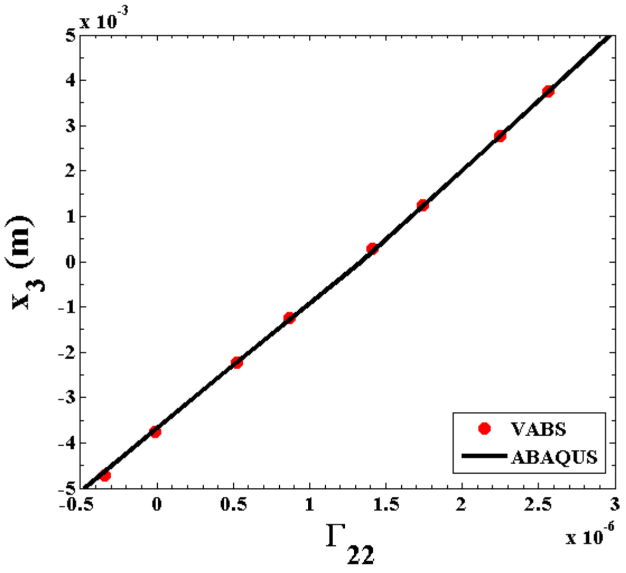

Figure 7: Strain distribution in x_2 direction along the thickness

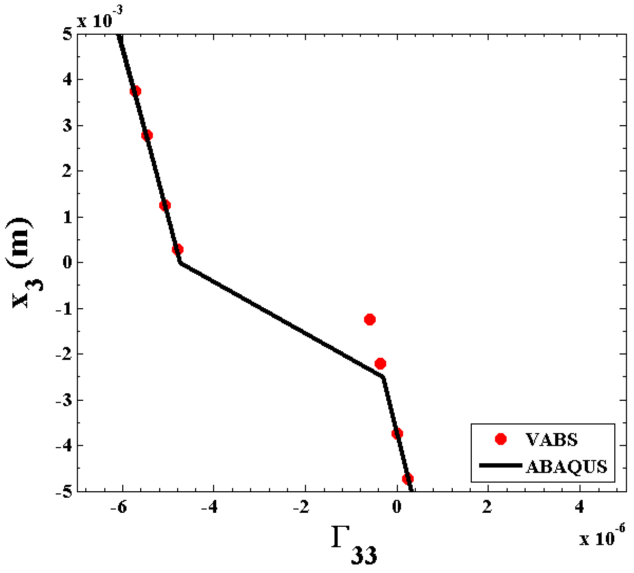

Figure 8: Strain distribution in x_3 direction along the thickness

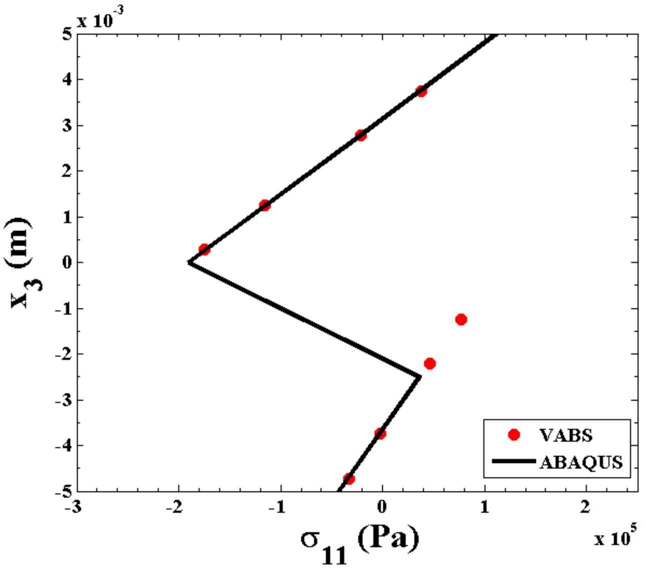

Figure 9: Stress distribution in x_1 direction along the thickness

4 Conclusions

An asymptotically correct patch PZT model has been developed and validated with the simulation results obtained in ABAQUS. The methodology used provides a modeling framework with accuracy as good as a 3D analysis but with significantly lower computational cost and time, as advantageous in a reduced model. The work here also projects the recovery of the field variables, which shows good agreement with the 3D ABAQUS results. The patch actuator model developed is quite capable of predicting the mechanical field variables in all the three planes with an electrical input voltage. The recovery results have shown good agreement with the ABAQUS.

Hence we can summarize the main features of the proposed model :

a. An asymptotically correct classical PZT Patch actuator model has been developed by the methodology named Variational Asymptotic Method (VAM).
b. The theory is entirely devoid of any assumptions made on distribution of mechanical and electrical field inside the structure.
c. Recovery of the field variables of a patch actuator model and its validation is a major novelty of the proposed work.

Acknowledgement

The authors would like to thank the department of Applied Mechanics, IIT-Delhi for their encouragement and Ministry of Human Resource and Development (MHRD), India, for their financial support.

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