Link Shape Optimization for Input Torque Reduction

Vinay Gupta, Subir K. Saha, Himanshu Chaudhary

Abstract

The reduction of input torque in robots can be performed by optimal distribution of mass in the links. Conventionally, optimal distribution of the mass is carried by adding counter weights, etc. Addition of counterweights increases the overall mass of the system. As a result torque reduction at each joint is not trivial. This paper proposed a methodology for achieving optimal mass distribution in link by changing its shape rather than adding counterweights. The problem formulated here is a shape optimization of the links of robots for the desired mass inertia properties. These desired inertia properties are the outcome of the optimization routine run for the minimization of actuating torques at the joints of the robot. This approach for the shape optimization is based on the material distribution method, which is mainly used for the topology and shape optimization of structures.

Keywords: Shape optimization, torque minimization, manipulator

1 Introduction

Torque minimization through balancing can results in reduced noise, vibration and increased fatigue life of the components. Moreover, small driving motors consume less power and hence good from economic point of view. The large amount of energy can be saved by designing the robots for minimum driving torque. Balancing can be categorized as active and passive methods [1]. In active balancing additional devices such as cams, gears, springs, balancing etc are used. These additional devices add complexities and mass to the system. Hence, reducing the input torques at all the joints is not trivial. The passive balancing is done by internal mass redistribution by adding counterweights or by changing the shape of the link. The counter weight also adds mass to the system; hence shape optimization of the link is preferred in this paper. The shape optimization meant here as to find the shape of the link for the desired or optimized mass, mass center location and mass moment of inertia about center of mass. The optimized inertia properties determined for minimum torque using equivalent model of point-masses [2].

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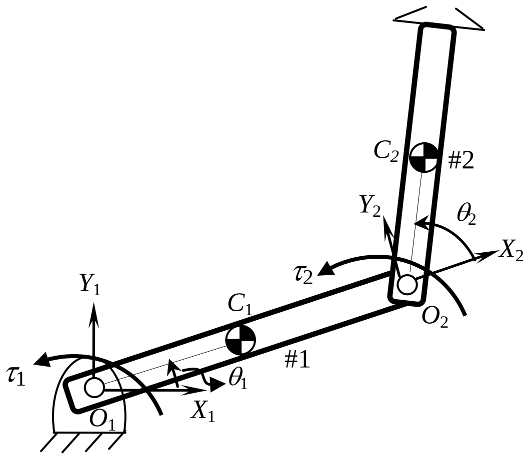

Figure 1: A two-link robot arm

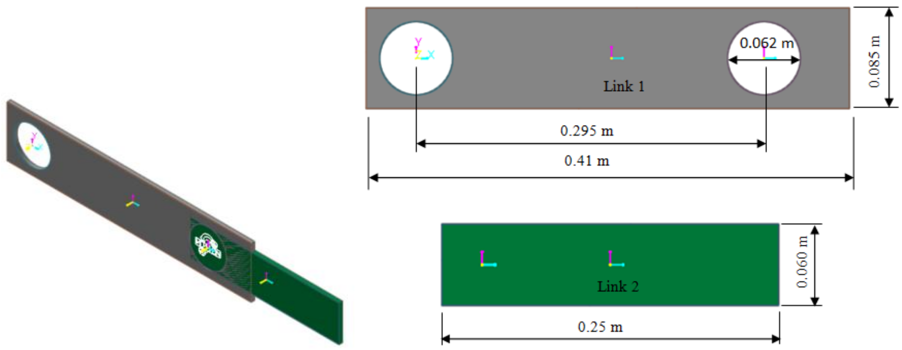

Figure 2: RecurDyn model of links

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The shape optimization of the links is not common, due to its difficulty. Different domain knowledge, e.g., from CAD, FEM etc., are required for shape optimization. Moreover, its practical realization is also not easy. The challenge is that whether the designed link can be manufactured. To the best of author's knowledge very few literatures are available on to find the shape for the given mass and inertia properties. Recently in [3] the shape synthesis of the links of planar closed-loop mechanism is performed to minimize shaking force and shaking moment. The shape synthesis is done by changing the shape of the boundary. The links boundary was parameterized by B-splines curve and their control points are considered as the design variables. In [4] the shapes of the links of slider crank mechanism by varying the boundary. In [5] small element superimposing (SSE) method was used for the shape optimization of coupler of an RSSR mechanism. The coupler was designed to minimize shaking force and shaking moment of the four bar mechanism. The paper did not provide much detail how the methodology led to particular shape of the coupler. In [6], the shapes of the counterweights were optimized to balance the shaking force and shaking moment using the voxel-based discretization. This method was motivated by the discretization used in topology optimization of structures. The synthesis of planar link's geometry was presented in [7] for the desired inertia properties. The methodology was based on control topology transformation. The paper did not discuss the effect of synthesised geometry on the dynamics. In [8], link shape optimization is formulated as material distribution problem. The methodology inspired from material distribution method used for the topology optimization of the structures. In this approach, a body was taken as initial design and discreitized in to square or rectangular elements. An artificial material density is assigned to each element and these densities are identified as design variables. The value of these densities is allowed to vary between 0 and 1. 0 corresponds that the element is removed. The practicality of material distribution approach is first explored in [10]. The advantage of this approach is the fixed FEM discretization as in sizing optimization for the entire iterative process and any shape can be approximated with this method. The practical aspect such has holes in the link for the connection can also be planned easily. On the other hand, boundary variation techniques [10, 11] for shape optimization required re-meshing several times during the optimization and limited to change in the boundary only. Proper techniques are required for re-meshing otherwise it results in non-smooth shapes. The methodology is illustrated with two-link planar robot.

The paper is organized as follows: Section 2 describes the robots architecture, Sections 3 presents shape optimization scheme, and Section 4 concludes the paper.

2 Model Description

Figure (1) shows schematic diagram of the two-link planar robot arm. Figure (2) shows the RecurDyn [12] model of the robot chosen for the purpose of validation of the MATLAB results [13]. The link dimensions and topology are selected such that the practical aspects can also be taken into account. For example, holes in the first link were made, as shown in Fig. (3), so that one can fix bearing and mount motor shaft to rotate the link. Also, as a future work the optimum design can be tested on the two-link planar robot available at the Program for Autonomous Robotics (PAR) Lab, IIT Delhi. The links are flat rectangular bar of uniform thickness 't_i', where 'i' represents the link number. Table 1 gives the desired mass and inertia properties of

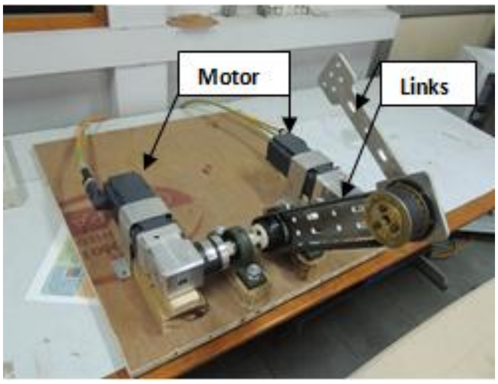


Figure 3: Two-link planar robot arm at PAR lab, IIT Delhi

Table 1: Desired mass and inertia properties

Link (i)	m_i (kg)	(a_i) (m)	I_i^c (kg-m^2)	t_i (m)	r_i (m)	ψ_i (deg)
1	1.357	0.295	0.0088	0.006	0.2676	-179.18
2	0.7065	0.205	0.0019	0.006	0.1824	173.13

the robot at hand. These mass and inertia properties are obtained from the torque minimization carried out for the robot at hand. Where m_i is the mass, a_i is the link length measured between two consecutive origins where links are hinged to each other. For example a_1 is length of link 1, measured between O_1 and O_2. I_i^c is the mass moment of inertia about center of mass of the links, r_i and ψ_i are the polar coordinates of mass center from the body fix frame. As per the DH frames adopted in this paper, the body fix frame is attached at the tip of the link, i.e., $X_2Y_2Z_2$, is attached to link1, whereas τ_i is the torque at the joint and θ_i is the relative joint angle between the links. C_i is the mass center location.

3 Shape Optimization

The shape optimization of the link is solved as optimum material distribution problem. This process involves some pre and post processing. The pre-processing is the fixed discretization of the initial body, numbering each element, an example is shown in Fig. (4).

3.1 Objective function

The objective function was based on the desired mass inertia properties, i.e., desired mass, mass center location and mass moment of inertia about center of mass. The problem was treated as single objective function. The square of the error of the moment of inertia and mass center location was minimized. The desired mass was taken care by adding constraints. The problem is proposed as

$$\underbrace{\text{minimize}}_{\rho} \quad f = (I_{zz}^c - I_{zz}^{c*})^2 + (\bar{x} - x^*)^2 + (\bar{y} - y^*)^2 \tag{1}$$

where I_{zz}^c, $\bar{x}, \bar{y}$ are the moment of inertia about center of mass, and mass center coordinate respectively. With the '*' mark are the desired parameters respectively. The moment of inertia and mass center location are evaluated as

$$I_{zz}^c = \sum_{k=1}^{n} A_k t_k \rho_k \{(l_k^2 + b_k^2)/12 + ((x_k - \bar{x})^2 + (y_k - \bar{y})^2)\} \tag{2a}$$

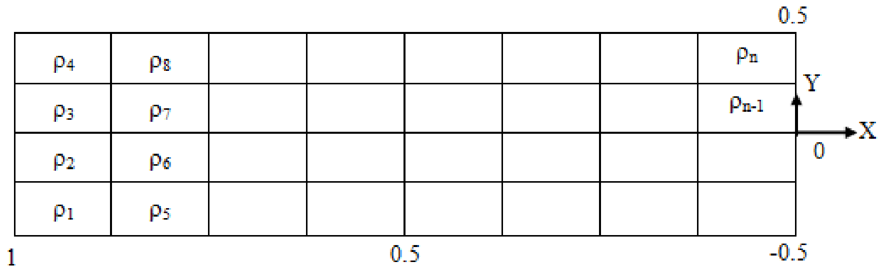


Figure 4: Illustration of the elements

$$\text{where, } \bar{x} = \frac{\sum_{k=1}^{n} t_k A_k \rho_k x_k}{\sum_{k=1}^{n} t_k A_k \rho_k} \; ; \; \bar{y} = \frac{\sum_{k=1}^{n} t_k A_k \rho_k y_k}{\sum_{k=1}^{n} t_k A_k \rho_k} \tag{2b}$$

A_k being the area, and x_k , y_k are mass center locations, and l_k, b_k are length and width of the of the k^{th} element. n being the number of elements.

3.2 Design variables and constraints

The design variable was the artificial density of the material denoted by ‘ρ’. The design variables were equal to the number of elements. The density was allowed to vary between 0 and 1. The initial value of artificial density or design variables was taken as 1 for each element. These imply that the initial link is complete body and shape of the link for the desired mass and inertia properties is achieved by removing the material. The constraints in Eq. (2a) ensures the mass of the link. The constraint in Eqs. (2b-2c) can be used if material is to be kept or removed from certain place. For example, to keep the hole in link 1 the material should be necessarily removed from that place.

$$\sum_{i=1}^{n} \rho_i = m_i^* \tag{2a}$$

$$\rho_i = 1; \tag{2b}$$

$$\rho_i = 0; \tag{2c}$$

where m_i^* is the desired mass of the link scaled to number of elements.

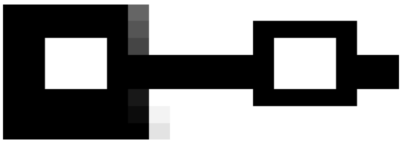


Figure 5a: Optimized shape of link 1

Figure 5b: Optimized shape of link 2

3.3 Steps for shape optimization

1. The mass of initial link is chosen more than the desired one. This is for the reason that the shape is achieved by removing the material.
2. Decide the number of elements for the discretization. The number of elements depends on the length and width of the link. It is free to choose the number of elements on one side, i.e., length or width. Other will be chosen accordingly.
3. Each element was assigned with an artificial density. These densities are the design variables for the shape optimization and allowed to vary between 0 and 1.
4. The desired and the actual parameters used in the shape optimization were scaled as per the initial link length as shown in Fig. (4).
5. The algorithm was written to calculate the mass center location and moment of inertia about the mass center using Eqs. (2a-2b).
6. Imposed the lower and upper bounds on the densities, along with the constraints on the mass.
7. The optimization algorithm 'fmincon' of 'MATLAB was used in this paper. One may use other algorithm too.
8. Plot the optimized design variables to see the shape of the link. 'Imagesc' of MATLAB was commonly used to display gray scale images.

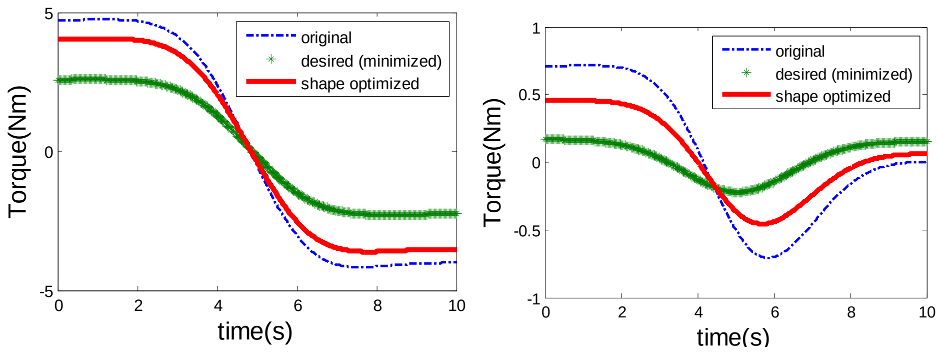
3.4 Link shapes

The links 1 and 2 were discreitized with 152 and 105 elements, respectively. The initial link width was taken double the actual width as mention in section 3.3. To ensure the link length, the density of the elements of the two rows i.e., center rows are constraint to be 1. Also to ensure the holes, the density of elements in the area of holes were taken as 0. The shape of the links after optimization is shown in the Fig. (5). Although no direct arrangement were made to avoid intermediate densities and discontinuity in the shape, the results shows no discontinuity. Also the intermediate densities are very less and can be approximated easily. The black color indicates the

Table 2: Mass and inertia properties of optimized shape

Link (*i*)	m_i (*kg*)	I_i^c (kg-m^2)	r_i (*m*)	ψ_i (deg)
MATLAB				
1	1.357	0.020	0.179	180
2	0.7065	0.0033	0.139	176.45
Shape built in RecurDyn				
1	1.683	0.024	0.181	177
2	0.737	0.004	0.143	174.4

density value as 1, other colors are some intermediate value of density. The Fig. (6) shows the comparison of the torques between original, desired (torque minimized) and optimized shape mass and inertia properties. The plot depicts that there is some compromise in torque reduction with optimized shape as exact mass and inertia properties could not be attained due to feasibility of link shape. The little saving of energy can also be beneficial for the robots working almost continuously over several months. The RecurDyn model shown in fig. (7), was attempted to be built using the optimized shape to simulate the situation of a practical fabrication. Table 2 shows the comparison of the realized mass and inertia properties with those obtained from shape optimization.



Joint 1 Joint 2

Figure 6: Comparison of torques

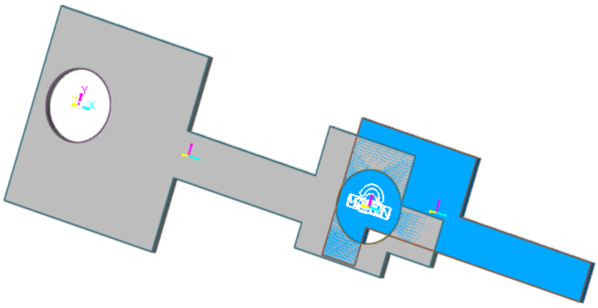


Figure 7: RecurDyn model of shape optimized robot

4. Conclusions

The paper discussed the methodology for shape optimization of the links of a planar robot arm. The shape optimization problem was formulated as material distribution problem which is inspired form the structural optimization. The methodology finds to be competent to find the shape of the links for the desired mass and inertia properties with some approximation. The biggest challenge of the shape optimization is the practical solution which was achieved for the case discussed in the paper. The methodology was demonstrated for a planar link and can be easily extended to the links in spatial motion. This will be reported in future.

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References

[1] I. S. Kochev, "General method for active balancing of combined shaking moment and torque fluctuation in planar linkages," Mechanism and Machine Theory, vol. 25, pp. 679-687, 1990.
[2] V. Gupta, H. Chaudhary, and S. K. Saha, "Dynamics and actuating torque optimization of planar robots," Joint International Conference ACMD/IMSD, pp. 1-10, 2014.
[3] K. Chaudhary, and H. Chaudhary, "Optimal dynamic balancing and shape synthesis of links in planar mechanism," Mechanism and Machine Theory, vol. 93, pp. 127-146, 2015.
[4] H. Azegami, L. Zhou, K. Umemura, and N. Kondo, "Shape optimization for a link mechanism," Struc. Multidisci. Optim., vol. 48, pp. 115-125, 2013.
[5] B. Feng, N. Morita, T. Torii, and S. Yoshida, "Optimum balancing of shaking force and shaking moment for spatial RSSR mechanism using genetic algorithm," JSME International Journal, vol. 43, pp. 691-696, 2000.

[6] M. Verschuure, B. Demeulenaere, E. Aertbelien, J. Swevers, and J. De Schutter, "Optimal counterweight balancing of spatial mechanisms using voxel-based discretization," proceeding of ISMA, pp. 2159-2173, 2008.
[7] G. Yoganand, and D. Sen, "Link geometry synthesis for prescribed inertia," National Conference on Machine and Mechanisms, S. Bandyopadhayay, and G, Sravan Kumar (Eds.), pp. 32-40, 2012
[8] M. P. Bendsoe, "Optimal shape design as a material distribution problem," Structural Optimization, vol. 1, pp. 193-202, 1989.
[9] M. P. Bendsoe, N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," Comput. Meth. Appl. Mech. Engg., vol. 71, pp. 197-224, 1988.
[10] Y. Ding, "Shape optimization of structures: a literature survey," Comp. Struct. Vol. 24, pp. 985-1004, 1986.
[11] R. T. Haftka, R. V. Gandhi, "Structural shape optimization-a survey," Comp. Meth. Appl. Mech. Engg. Vol. 57, pp. 91-106, 1986.
[12] RecurDyn, 2013, Version 8, Release 2. www.functionbay.de
[13] Matlab, 2013, MATLAB-The Language of Technical Computing, Version 8.1, Release 13a www.mathworks.in/products/matlab/