# A general purpose program for kinematic analysis of plane mechanisms

Suman Basak, Sumanta Neogy, Arghya Nandi

**Abstract**

Powerful packages based on multi body dynamics can solve virtually any dynamic system. But to the user they are black boxes. Kinematic analysis of plane mechanisms is vital to mechanical engineering. This analysis hardly requires such versatile tools. Further blind usage of these powerful tools does not permit the user to develop insight into the mechanisms. On the other hand packages based on simple but modular approach is ideally suited for the purpose. The present work has attempted to develop such a program for the purpose.

**Keywords:** Kinematic analysis of plane mechanism, Simple theory, General package, Modular approach, Four modules, Sequential calling, Building complex Mechanism

## 1 Introduction

A plane mechanism consists of a set of inter-connected rigid bodies in plane motion. Frontiers of computational dynamics have led to development of versatile packages which can simulate virtually any combination of bodies in general space motion. Such powerful packages use the theoretical generalization of multi body dynamics. The users, especially beginners, have little understanding of their algorithms. Many a time to solve simple mechanisms such packages are utilized which can best be thought of as underutilization of their capacity. What is more detrimental is that the user gets no insight into the logical building of the mechanism. The proposed work presents an alternative approach. The theory on which the work is based uses simple kinematics of particles and rigid bodies discussed at the undergraduate level. It decomposes the kinematic analysis problem of plane mechanism into four simple modules. The modules are rigid body, combination of rigid body and/or particles. These modules can also be thought of as truncated portions of mechanisms. The user needs to sequentially breakdown any plane mechanism into some or all of these modules. So, the advantage obtained in the study of kinematics of mechanism is that now the mechanism can be thought of combination of these basic units instead of

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rigid bodies and particles which happens to be the approach in the study of Engineering Mechanics. So, this technique is considerably beneficial to a mechanical engineer as he can now quickly develop complex mechanisms from these units. Further, the algorithm and theory behind the computational process is completely transparent. The presented combination has the beauty of decoupling the analysis process which means that the units can be independently solved. Some portions of this modular approach has been pointed out by Shu and Radcliffe (1) but the possibility of developing a general purpose package has not been explored here. Computer aided kinematic and dynamic analysis of mechanical systems has been done by Haug (2). Norton (3) has also used computational methods for analysis of mechanisms. Excellent discussion on mechanism analysis is also found in Mabie and Reinholtz (4), Sigley and Wicker (5) and Ramamurti (6). While the former two has not discussed on computational methods, the later has used specialpurpose program for analysis of mechanisms. Kinzel and Chang (7) has extended the work of Shu and Radcliffe (1) to mechanisms with slider input and has also coupled Goodman's inversion technique with the modular approach. Nagarajan A. and Bandyopadhyay, S. (8) has discussed a general framework for analysis of mechanisms. However, in the present work, a platform independent programming structure has been presented for developing a general purpose package for kinematic analysis of mechanisms. The program structure has a strong similarity with structure of FE programs which most users are already acquainted with. The structure is such that it can, with little modification, be extended to kinetic analysis.

# 2 Modules & Analysis:

The present section describes the four modules in terms of input and output variables of position and motion analysis, the basic principle & the final equation programmed and then discusses how a fairly complex mechanism can be analyzed using these modules. The inputs, for a module, are either given as input or are obtained as the output of some other module.

## 2.1 Modules:

### 2.1.1 Rigid body:

A rigid body (fig 1),in Plane mechanism, has 3 degrees of freedom. Commonly, they are the position (x & y co-ordinate) of an arbitrary point (defining point) and the angular position (angle $\theta$) of an arbitrary line (defining line) and are inputs to the module. The module positions the rigid body in the mechanism and so its output are the position of all points defined and the angular position of all lines defined.
The basic equation used is

$$x_2 = x_1 + r*\cos(\theta+\phi)$$
$$y_2 = y_1 + r*\sin(\theta+\phi) \qquad (1)$$

where point 1 is defining key point and point 2 is defined point.
r is the radial distance of definedpoint,
$\theta$ is the input angular position of defining line,

$\phi$ is the angle of the defined line w.r.t the defining line.
For position analysis, this calculation is repeated in a loop for the all defined lines. Motion (velocity and acceleration) analysis has the same approach. Linear motion of the defining point and angular motion of the defining line is the input to the module. The module calculates linear motion of the defined point. The basic equations used are

$$\dot{p}_3 = \dot{p}_1 + \dot{\theta} * (p_3 - p_1)$$
$$\ddot{p}_3 = \dot{p}_1 + \ddot{\theta} * (p_3 - p_1) + \dot{\theta} * (\dot{\theta} * (p_3 - p_1)) \quad (2)$$

where symbols have their standard meaning
The scalar component of the vector equations are used in the program.

### 2.1.2 Two link dyad:

The two link dyad (fig 2)consists of two rigid bodies connected together by a hinge joint at point 3. At point 1 and point 2, the dyad is connected to some other module by hinge. Once the module is solved, a general rigid body can be superimposed on either of the 2 line description of the rigid body to get motion of all defined points. The two link dyad has 4 degree of freedom. In position analysis the 4 variables to be specified as input are the x & y co-ordinate of point 1 and point 2. Further, from geometry of each rigid body, length $r_1$ and $r_2$ are known. Using this data, it isfirst checked whether the dyad can be assembled and then the angles $\theta_1$&$\theta_2$and the position of the point 3are calculated. The calculation process is simple as from position of point 1 and 2, 'd' is calculated. The assembly criterion of the dyad is d <$(r_1+r_2)$ & d>$|r_1 - r_2|$ . Then $\phi$ is calculated & using cosine rule of triangles, $\alpha$ is obtained. $\theta_1$is obtained using $\theta_1=\phi\pm\alpha$ where +/- is to be chosen depending upon the orientation of the dyad. Co-ordinate of point 3 is calculated from position of point 1 &$\theta_1$and finally$\theta_2$ is obtained from calculated position of point 3 and given position of point 2. These simple steps are followed in position analysis.Motion (velocity and acceleration) analysis has the same approach. The inputs of the module are the velocity and acceleration of point 1 & 2. And the output of the module is the angular velocity and angular acceleration of both the rigid bodies and the velocity and acceleration of point 3. The basic equations used are

$$\dot{p}_3 = \dot{p}_1 + \dot{\theta}_1 * (p_3 - p_1) = \dot{p}_2 + \dot{\theta}_2 * (p_3 - p_2)$$
$$\ddot{p}_3 = \dot{p}_1 + \ddot{\theta}_1 * (p_3 - p_1) + \dot{\theta}_1 * (\dot{\theta}_1 * (p_3 - p_1)) \quad (3)$$

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### 2.1.3 Oscillating slider:

The oscillating slider consists of a rigid body & a second body- modeled as a particle which is constrained to move along a straight line arbitrarily located in the former rigid body. The module may be connected to other modules by hinges at3 points. Point 1 is arbitrarily located in the rigid body(has eccentricity 'e' with the guide), point 2 is the sliding particle (called slider) and point 3 is a point in the rigid body collinear with the guide. Once the solution is completed, a rigid body can be superimposed with the information of point 1 and point 3 to obtain the motion of a point in the rigid body.

This module has 4 degree of freedom. In position analysis the 4 variables specified as inputare position of point 1 and absolute position of point 2 (x & y co-ordinate of each point). Using this data, it is first ascertained whether the oscillating slider can be assembled and then the angular position of the rigid body, the relative position of point 2 and the absolute position of point 3 is calculated. The calculation process is simple. ‘d’ is calculated& from absolute position of point 1 & 2. The module can be assembled if $d^2 > e^2$ with ‘e’ known, $r_2$ is calculated. $\phi$&$\alpha$ are then calculated.$\theta$is obtained as $\phi\pm\alpha$ depending upon the configuration of the module. The position of the point 3 is specified (input) by $r_3$. Using $r_2$, $r_3$&$\theta$, it is easy to calculate x & y co-ordinate of point 3. These simple steps are put in the position analysis routine.Motion (velocity & acceleration) analysis has the same approach. The input of the module is the absolute velocity & acceleration of point 1 & 2. The output of the module is the angular velocity & acceleration of the rigid body, the relative velocity & acceleration of point 2 and the absolute velocity and acceleration of point 3.

The basic equations used are –

$$\dot{p}_2= \dot{p}_1+\dot{\theta} *(p_2-p_1)+\dot{r}_2*\hat{r}_2$$
$$\ddot{p}_2= \dot{p}_1+\ddot{\theta}*(p_2-p_1)+\dot{\theta} * (\dot{\theta} *(p_2-p_1)) + \ddot{r}_2*\hat{r}_2 + 2*\dot{\theta} *\dot{r}_2*\hat{r}_2 \qquad (4)$$

The scalar forms of these equations are directly incorporated in the program.

### 2.1.4 Rotating guide:

The rotating guide consists of 2 rigid bodies and a third body modeled as a particle. The particle is hinged to one of the rigid bodies and it is constrained to move along a straight line arbitrarily located in the other rigid body.Once the module is solved, the rigid body model can be superimposed on either of the line description of the rigid body to get motion of any defined point on the rigid body.

The module has 5 degree of freedom.In position analysis the variables to be specified as input are the position (x & y co-ordinate) of points 1 & 2 and angular position of the rigid body contains the slot. This is either given as input or obtained as output of some other module. Further from geometry of the former rigid body if length $r_1$ is known, it is possible to first check whether the rotating guide can be assembled and then the module gives as output the angular position of the former rigid body and the absolute and relative position of the sliding particle. The calculation process is simple as from position of point 1 & 2, ‘d’& ‘$\phi$’ are calculated. Using the distance criterion between point 1 & 3, it is possible to judge whether the module can be assembled. In case it can be assembled relative position ($r_2$) can be calculated & two values of $r_2$ are possible which decides the orientation. Using the angular position of the rigid body and the value of $r_2$ , the co-ordinate of point 3 can be obtained.Motion (velocity & acceleration) analysis has the same approach. The input of the module is velocity and acceleration of point 1 & point 2 and the angular velocity and angular acceleration of the rigid body carrying the slider. The output of the module is the angular velocity & angular acceleration of the other rigid body and absolute & relative velocity and absolute & relative acceleration of the slider.

The basic equations used are

$$
\begin{aligned}
\dot{p}_3 &= \dot{p}_1+\dot{\theta}_1*(p_3-p_1)\\
&= \dot{p}_2+\dot{\beta}*(p_3-p_2)+\dot{r}_2*\hat{r}_2\\
\ddot{p}_3 &= \dot{p}_1+\ddot{\theta}_1*(p_3-p_1)+\dot{\theta}_1*(\dot{\theta}_1*(p_3-p_1))\\
&= \ddot{p}_2+\ddot{\beta}*(p_3-p_2)+\dot{\beta}*(\dot{\beta}*(p_3-p_2))+\ddot{r}_2*\hat{r}_2+2*\dot{\beta}*\dot{r}_2*\hat{r}_2 \qquad (5)
\end{aligned}
$$

Scalar component of the equation are used to obtain $\dot{\theta}_1$, $\dot{r}_1$&$\ddot{\theta}_1$,$\ddot{r}_1$ which are incorporated in the program.

Thereafter absolute motion of point 3 is obtained using-

$$\dot{p}_3=\dot{p}_1+\dot{\theta}_1*(p_3-p_1) \qquad (6)$$

The input & output parameters of each module are summarized below in tabular form:

Table 1 Table of input and output parameters of the four modules

| **Module** | **Input** | **Output** |
|---|---|---|
| **Rigid body** | Defining point position & motion<br>Define line position & motion | Position & motion of any other point<br>Position & motion of any other line |
| **Dyad** | Position & motion of end points 1 & 2, link lengths | Position & motion of point 3, angular position & motion of either rigid body of the dyad |
| **Oscillating slider** | Absolute Position & motion of points 1 & 2 and eccentricity(e) | Relative Position & motion of point 2, angular position & motion of rigid body, position & motion of point 3 |
| **Rotating guide** | Absolute position & motion of point 1& 2, angular position & motion of rigid body containing the guide, crank length | Relative & absolute position & motion of slider. |

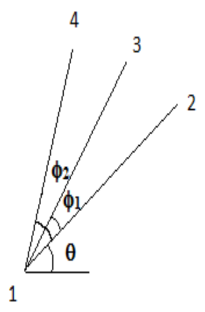


Figure 1

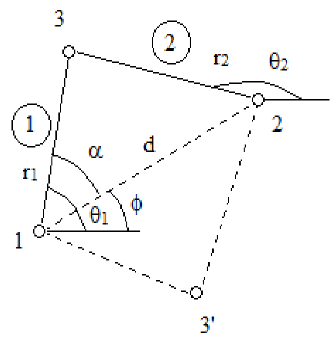


Figure 2

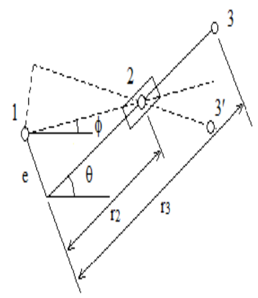


Figure 3

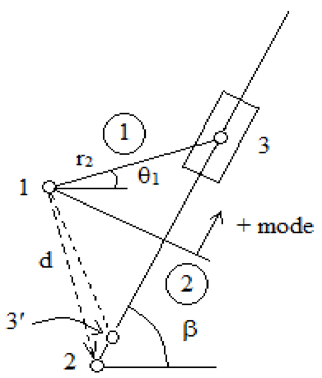


Figure 4

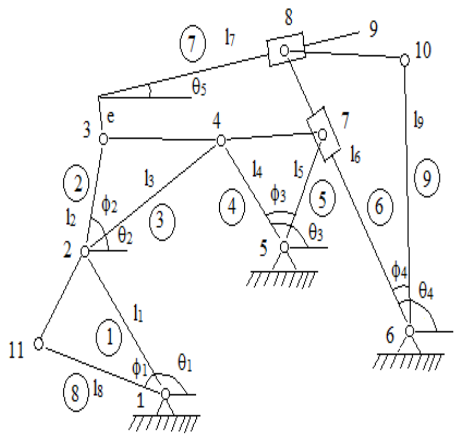


Figure 5

# 3 Inputs to program:

The input to the general purpose program developed is described in the present section using the complex mechanism shown in fig 5.

Figure 5 is an example of a single input (i.e. single dof) fairly complex plane mechanism. The point nos and line nos are shown in the figure. Point 1, 5 & 6 are fixed and their positions are given. $\theta_1$ and motion of crank 1-2 is given. $\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$ which define the geometry for the 4 rigid bodies and 'e' is given. The analysis finds out position & motion at points 10,11,9, sliding motion of sliders 7,8 and angular motion of cranks 4-5, 6-8.

The inputs are classified into –

## 3.1 General input:

The input parameters for the overall mechanism are specified here.

i) The number of points and number of lines in the plane mechanism need to be specified. For the complex mechanism shown in the figure 5 –
Number of points = 11
Number of lines = 9

ii) The number of fixed points as well as their index and coordinates need to be specified.
Number of fixed points = 3

Coordinate table of fixed points:

| Fixed point index | x | y |
|---|---|---|
| 1 | $x_1$ | $y_1$ |
| 5 | $x_5$ | $y_5$ |
| 6 | $x_6$ | $y_6$ |

iii) The number of rotating input as well as their index, angular position, angular velocity and angular acceleration are to be specified.
Number of rotating inputs = 1

| Line index | θ | ω | α |
|---|---|---|---|
| (1) | $\theta_1$ | $\omega_1$ ($\dot{\theta}_1$) | $\alpha_1$ ($\ddot{\theta}_1$) |

## 3.2 Modular input:

The modular input gives the modules into which the mechanism is to be split. It also gives relevant input parameters for each module. Further the modular inputs should be specified in the order in which the analysis flows. This is described with the help of the complex mechanism. Line length and angle are shown as variables as shown in the fig. (5)

No. of blocks required is taken under the parameter 'nbloc'

**i) Rigid body:**

| ITYP | $n_p$ | Point index | | | Line index | | Line length | | Angle of line | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 (rigid body) | 2 | 1 | 2 | 11 | 1 | 8 | $l_1$ | $l_8$ | 0 | $\phi_1$ |

**ii) Two link dyad**

| ITYP | Point index | | | Line index | | 1st link length | 2nd link length | Parity |
|---|---|---|---|---|---|---|---|---|
| 2 (dyad) | 2 | 5 | 4 | 3 | 4 | $l_3$ | $l_4$ | +1 |

**iii) Rigid body**

| ITYP | $n_p$ | Point index | | | Line index | | Line length | | Angle of line | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 3 | 4 | 2 | 3 | $l_3$ | $l_2$ | 0 | $\phi_2$ |
| 1 | 3 | 5 | 4 | 7 | 4 | 5 | $l_4$ | $l_5$ | 0 | $-\phi_3$ |

**iv) Oscillating Slider**

| ITYP | Point index | | | Line index | Eccentricity | Line length | Parity |
|---|---|---|---|---|---|---|---|
| 3 | 6 | 7 | 8 | 6 | 0 | $l_6$ | +1 |

**v) Rigid body**

| ITYP | $n_p$ | Point index | | | Line index | | Line length | | Angle of line | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 (rigid body) | 2 | 6 | 8 | 10 | 6 | 9 | $l_6$ | $l_9$ | 0 | $-\phi_4$ |

**vi) Oscillating slider**

| ITYP | Point index | | | Line index | Eccentricity | Line length | Parity |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 8 | 9 | 7 | 0 | $l_7$ | -1 |

The example does not need the rotating guide module. So, a blank format for entry of Data to this module is shown below

vii) **Rotating Guide:**

| ITYP | Point index | | | Line index | | Length of rotating bar | angular position of guide link | Parity |
|---|---|---|---|---|---|---|---|---|
| 4 | | | | | | | | |

# 4 Program Structure:

The structure in which the program has been developed is shown in the form of an overall flow diagram in figure 6.

In, the figure, the arrays {X}, {VX}, {AX} & {θ}, {ω}, {α} are initialized by large numbers. As the position, velocity and acceleration analysis proceeds with proper sequential order, the arrays are filled up and at the end the complete array is determined. The array contains the position, velocity, acceleration of all defined points and angular position, angular velocity, angular acceleration of all defined lines. In case a large number is retained in the array, it implies that the said variable has not been properly calculated either because of erroneous input or error in program.

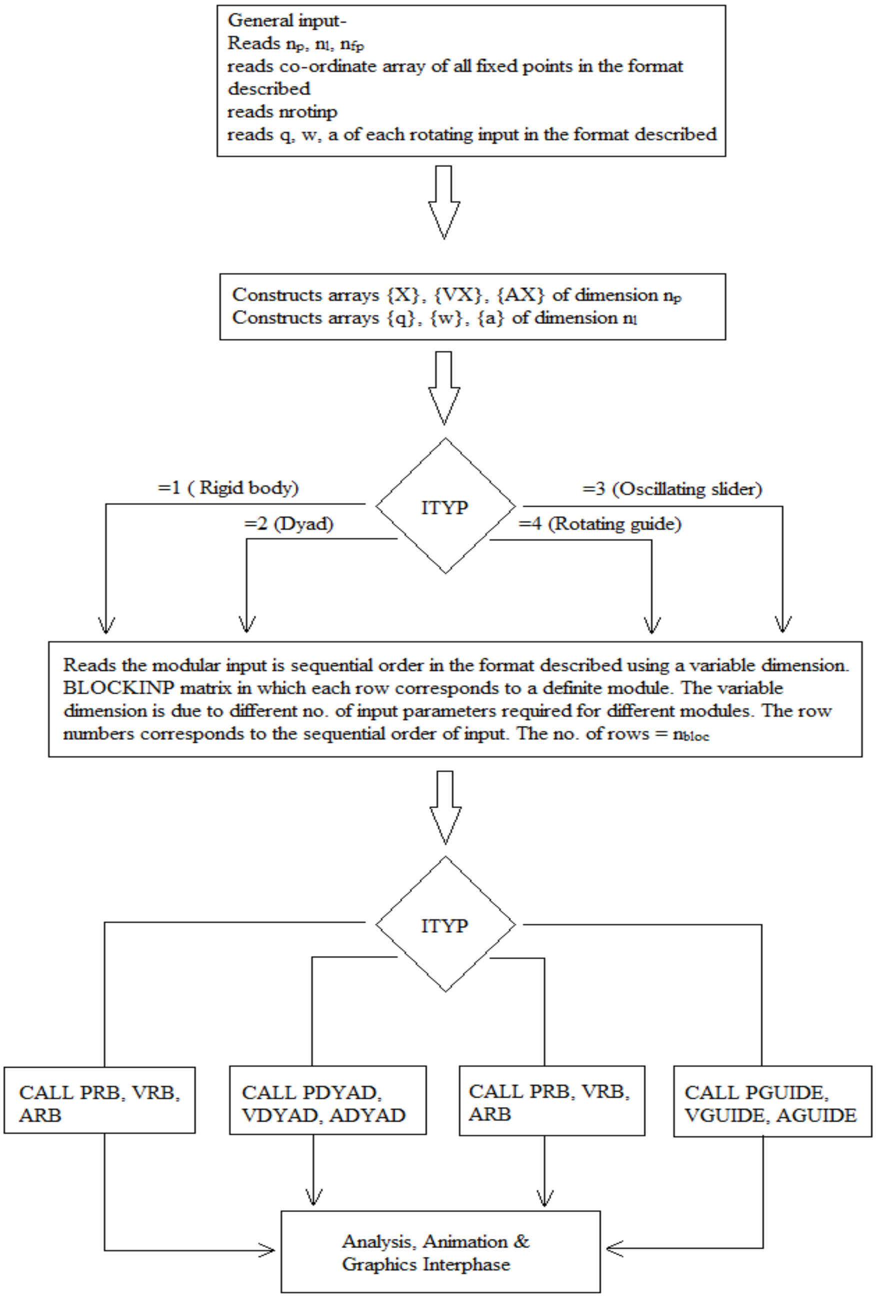


Figure: Program structure shown in flowchart

# 5 Example problem:

The complex mechanism discussed earlier is solved here.
Example The complex mechanism in figure 5 is completely solved here. The following data are taken as input (fig 5)

Lengths of members $l_1 = 6$, $l_2 = 60$, $l_3 = 8$, $l_4 = 4$, $l_5 = 4$, $l_6 = 10$, $l_7 = 10$, $l_8 = 6$, $l_9 = 8$; crank angle $\theta_1 = 120°$

Angular position of various lines $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 30°$
Angular velocity of crank (line 1) = 10 rad/s
The results are shown in Table 2 and Table 3.

Table 2 Linear position and motion of points of complex mechanism

| Point no | x(mm) | y(mm) | $v_x$(mm/s) | $v_y$(mm/s) | $a_x$(mm$^2$/s) | $a_y$(mm$^2$/s) |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | -3.0000 | 5.1962 | -51.9615 | -30.0000 | -0.0520 | 0.0030 |
| 3 | 0.2993 | 10.2076 | -54.4177 | -28.3830 | -0.0248 | -0.0011 |
| 4 | 4.1506 | 8.7834 | -53.7197 | -26.4954 | -0.0498 | 0.0102 |
| 5 | 5.9200 | 5.1960 | 0 | 0 | 0 | 0 |
| 6 | 11.9200 | 1.7320 | 0 | 0 | 0 | 0 |
| 7 | 7.2055 | 8.9838 | -56.7206 | 19.2505 | -0.1072 | 0.0037 |
| 8 | 6.4695 | 10.1160 | -35.9244 | -23.3548 | 5.4351 | 0.8394 |
| 9 | 10.4266 | 11.4064 | -54.2221 | -30.0360 | -1.3184 | 1.0916 |
| 10 | 11.4974 | 9.7208 | -34.2311 | -1.8109 | 0.4080 | 0.7990 |
| 11 | -5.1962 | 3.0000 | -30.0000 | -51.9615 | -0.0300 | 0.0052 |

Table 3 Angular position and motion of links of complex mechanism

| Line no | $\theta$ | $\omega$ | $\alpha$ |
|---|---|---|---|
| 1 | 2.0944 | 10.0000 | 0 |
| 2 | 0.9886 | 0.4901 | -0.0083 |
| 3 | 0.4650 | 0.4901 | -0.0083 |
| 4 | 2.0290 | 14.9746 | 0.0173 |
| 5 | 1.2436 | 14.9746 | 0.0173 |
| 6 | 2.1472 | 4.2849 | - |

| | | | |
|---|---|---|---|
| 7 | 0.3152 | -0.1632 | - |
| 8 | 2.6180 | 10.0000 | 0 |
| 9 | 1.6236 | 4.2849 | - |

# 6. Conclusion

The approach to analyse plane mechanisms as in modular form has been implemented here using the MATLAB platform. The modules have been described in a very general fashion which may help uncover the power in them. Prudently made combinations can help dynamically model any mechanism and as well make position and motion analysis of any point in the mechanism. The software developed is quite general in nature.

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