Modeling and Simulation of a Three-Joint Prosthetic Finger Actuated by Remaining Functional Natural Fingers: A Bond Graph Approach

Arvind Kumar Pathak, Neeraj Mishra, Anand Vaz

Abstract

This paper addresses issues pertaining to hand prostheses with functional finger joints actuated by remaining natural fingers. These offer improved options for rehabilitation of hand impairments with partial disabilities. A prosthetic finger mechanism with three joints, offering greater dexterity and based on the like-unlike configuration presented earlier is used to realize the implementation of the abstract concept of Opposition Space. A methodology for modeling and simulating the dynamics of a three-joint prosthetic finger actuated by remaining natural finger joints based on the string-tube mechanism using multibond graph is presented here. The numerical simulation of the Bond graph model is performed using MATLAB and SCILAB. 3D animation of the simulated results performed in the open source VRML environment has been used to obtain a better understanding through visualization.

Keywords: Margins, Tables, Figures, and Equations (to be formatted in 9pt Times New Roman)

1 Introduction

Hand prostheses with functional finger joints actuated by remaining natural fingers offer improved options for rehabilitation of hand impairments with partial disabilities. String-tube based mechanisms have been proposed earlier for joint actuation from...
natural active joints to passive prosthetic joints [1]–[3]. A prosthetic finger mechanism with three joints provides greater dexterity and hence more possibilities for further exploration of the like-unlike configuration which was presented earlier to realize the implementation of the abstract concept of Opposition Space [4], [5]. This paper presents a methodology for modeling and simulating the dynamics of a three-joint prosthetic finger actuated by remaining natural finger joints using the string-tube mechanism. The integration of the models of rigid phalange and prosthetic link subsystems with string-tube subsystems is explained in detail and modeled using multibond graph. The model is analyzed for power transactions and the perspective of cause and effect, a unique feature of Bond graph. The equations governing the dynamics of the system are derived from the Bond graph which also facilitates the coding for simulation [6]–[8]. The numerical simulations have been performed using MATLAB and SCILAB. Results of simulation are presented graphically and depict: the forces and torques required at every joint of the finger during performance of predefined tasks, linear and angular velocities, positions of centers of mass for each prosthetic link, angle turned by each link with time, extension in string-tubes and so on. 3D animation of the simulated results performed in the VRML environment has been used to obtain a better understanding through visualization [9].

Animation of any solid body in 3D requires information of: the position of its center of mass, axis of rotation about which it is going to be rotated and the angle of rotation by which it is going to be rotated with respect to time. Results coming from the bond graph simulation are the inputs for the animation of the mechanism.

This paper is organized as follows. The following subsections of the introduction give the basic fundamentals of the prosthesis and their assembly into a complete model. Section 2 provides a discussion on 3-D animation in VRML and its implementation in this paper. A discussion of simulation results for the multi-bond graphs based model is exhibited in section (3). The conclusion and potential for future work is explored in section (4).

1.1 Proposed prosthetic device

Basic configuration of the three finger mechanism which is going to be considered here is shown in Fig (1). The human finger joints are basically made out of rigid skeletal links bones, phalanges and metacarpals, which are generally pivoted at joints so as to give revolute movement. Inspite of the fact that a joint is not strictly constrained to a single degree of freedom, revolute motion about one axis usually prominent.

Incitation of the joints is performed by muscles that are associated with these skeletal links. In view of a similarity with the human instrument of joint incitation, the guideline can be shown as demonstrated in Fig. (2), utilizing a string-tube which is
called a Bowden link. It has two configuration which are organized as *like* and *unlike* configurations as per shown in Fig. (2).

![Diagram of three finger mechanism](image)

**Figure 1:** Basic configuration of three finger mechanism

These configurations are based on principle of opposition of space which was discussed in detail by the authors [2].

![Like and unlike configuration of active and passive joints](image)

**Figure 2:** *Like* and *unlike configuration* of active and passive joints

### 1.2 Modeling Using Multi Bond Graph

For an application like the hand prosthesis considered here, utilizing bond graph structure brings about an extensive representation. It is beneficial if the representation
is made smaller. Multi bond graph gives such a manifestation of minimal representation, while holding the other critical linguistic builds of scalar bond diagrams. BGOM of prosthetic finger consider (1) Rigid body dynamics (2) Orientation (3) Translational coupling (4) Rotational coupling (5) String tube mechanism (6) Virtual Reality Modeling Language.

### 1.2.1 Rigid body dynamics

Multibond graph modules are developed for fingers based on multi body dynamics in three dimensional space. Every phalange of the finger is thought to be a rigid body.

Fig. (3) Speaks to a bond graph for dynamics of a rigid link. It is characterize the joint impact of translational and rotational elements of the rigid link which can be derived from the Newton-Euler mathematical statements in inertial frame 0.

\[
\begin{align*}
\sum_i \mathbf{C} \mathbf{F}_i &= \mathbf{M} \\
\sum_i \mathbf{0} \mathbf{r}_i &= \mathbf{0} \\
\end{align*}
\]

\[
\begin{align*}
\sum_i \mathbf{C} \mathbf{r}_i &= \mathbf{0} \\
\sum_i \mathbf{0} \mathbf{r}_i &= \mathbf{0} \\
\end{align*}
\]

Similarly the angular momentum of the rigid link and resultant of the applied torques can be considered to be concentrated at center of mass of link in inertial frame 0.

\[
\begin{align*}
\sum_i \mathbf{C} \mathbf{r}_i &= \mathbf{0} \\
\sum_i \mathbf{0} \mathbf{r}_i &= \mathbf{0} \\
\end{align*}
\]
\[
\frac{d}{dt}\{I_0 B \times 0_{B} B\} = \sum_{i=1}^{n} C_i \times 0_{i} F_i + \sum_{j=1}^{k} 0_{j} \tau_j
\]  
\hspace{1cm} (2)

### 1.2.2 String tube mechanism

The prosthesis has two sides as, an active side which is associated with a natural finger and a passive side which is incited by the active side. The mechanics of the string-tube connection between an active and a passive joint is shown in the bond graph of Fig. 8. The two string-tubes are represented by the two paths between junctions \(1_{\theta_p}\) and \(1_{\theta_a}\).

\[
\frac{\dot{\theta}_A}{\dot{\theta}_p} = \frac{r_{1p}}{r_{1A}} = \frac{r_{2p}}{r_{2A}}
\]  
\hspace{1cm} (3)
Which is a constant ratio. By designing the ratio according to the requirement, necessary torque and movement of the active and passive joint can be achieved.
2 3-Dimensional Animation

Animation of any solid body in three dimensional space requires three things: position of centre of mass, axis of rotation about which the body is going to be rotated given by its unit vector and angle of rotation by which the body is going to be rotated with respect to time. Results coming from the simulation are the input for the animation of the mechanism. 3D animation of the simulated results have been performed in the VRML environment to obtain a better understanding through visualization. VRML is an open source software which is integrated to any simulation software for the animation.

2.1 VRML

The Virtual Reality Modeling Language (VRML) is an ISO standard that is open, text-based, and uses a WWW-oriented format used to define a virtual world that you can display with a VRML viewer. VRML has been designed to fill a variety of needs, including the ability to

- Animate objects in a scene
- Interact with the scene
- Control the scene with small programs, referred to as scripts, and
- Play sounds and movies within the scene.

So we have used VRML to develop a 3D animation of simulation of physical system. It is also able to control environmental condition inside the virtual world.

2.1.1 3-D Animation in VRML

There are two things required for performing 3-D animation in VRML, (a) Position of the center of mass, and (b) Unit vector and rotation angle

2.1.1.1 Position of center of mass

When the animation is performed, position of center of mass of that body is changed at every instant of time. To get the position of center of mass at every instant of time velocity of the object is integrated for a given interval of time

\[ \mathbf{r}(t) = \mathbf{r}(t_0) + \int_{t_0}^{t} \mathbf{v} \, dt \]

By integrating this equation, position vector of the object is going to be determined
2.1.1.2 Unit vector and rotation angle

2.1.1.2.1 Unit vector

The unit vector is the axis of rotation about which the body is going to be rotated. Unit vector for anybody is the ratio of the angular velocity if that body with respect to and expressed to inertial frame and modulus of the angular velocity if the same body.

$$\hat{\mathbf{w}}_B = \frac{\mathbf{w}_B}{\|\mathbf{w}_B\|}; \quad \|\mathbf{w}_B\| = \dot{\theta}$$

(5)

2.1.1.2.2 Rotation angle

Rotation angle is the angle by which the body is rotated about the axis of rotation. Since rotation angle is changed at every instant of time so for determining the rotation angle equation given below is going to be integrated for a given interval of time.

$$\theta(t) = \theta(t_i) + \int_{t_i}^{t} \hat{\mathbf{w}}_B \cdot \mathbf{w}_B d \xi$$

(6)

2.2 Steps for Performing 3D Animation in VRML

- Draw all individual component of any mechanism in any design software like solid works etc., having C.G. at the origin.
- Save them with .wrl extension
- Add all the component in VRML library.
Drag all components in a new file of VRML and allot specific node name to every components respectively. Follow all commands of VRML- MATLAB interface. VRML files describe 3D objects and worlds using a hierarchical scene graph. Entities in the scene graph are called nodes. flow diagram for the animation of the mechanism is represented in fig shown below.

![Flow Diagram](image)

**Figure 7: Steps for MATLAB - VRML interface**

### 2.2.1 Create a World Object in VRML

We begin by creating an object of class VRWORLD that represents the virtual world. The VRML file constituting the world was previously made using the V-Realm VRML builder contained in the Simulink 3D Animation product. The name of the file is VRMOUNT.WRL.

```matlab
World = vrworld ('vrmount.wrl');
```

### 2.2.2 Open and View the World in VRML

The world must be opened before it can be used. This is accomplished using the OPEN command.

```matlab
Open (world);
```

The virtual world can be viewed in the VRML viewer by command given below

```matlab
Fig = view (world, 'internal');
Vrdrawnow;
```

### 2.2.3 Accessing VRML Nodes

To access a VRML node, an appropriate VRNODE object must be created. The node is identified by its name and the world it belongs to.

```matlab
Node name = vrnode (world,'ABC');
```

Value required for the motion that is position of centre of mass, change in angular position (θ) and unit vector is provide to these nodes using specific commands given below.
Node name. Translation = (coordinates of position of centre of mass);
Where coordinates of position of centre of mass = X, Y, Z

Node name. Rotation = (axis of rotation, θ);
Where axis of rotation = three components of unit vector
All these concepts and principle are useful during the preformation of 3D animation in Virtual Reality Modeling Language.

3 Modeling and Simulation of three joint finger prosthetic mechanism

A complete string-tube based three-joint prosthetic finger system is demonstrated by gathering all elements as shown Fig. 8. The right part of the mechanism representing active link (natural finger) and left part representing the passive link (impaired link). The model pertains to general three-dimensional movement of the finger connects, and is not confined to a planar case just.

Figure 8: Three link prosthetic finger mechanism.
Torque is applied on active side and transferred to the passive side using string tube mechanism. Collaboration between every element of the mechanism is made utilizing ordinary multibonds or scalar bonds. One can show the motion of every last one of fingers of the hand, characteristic and prosthetic, utilizing modules as a part along these lines.

3.1 Simulation and discussion

3.1.1 Initial conditions and parameters for the simulation

The dynamics of a three joint actuated string-tube based prosthesis for an impaired hand has been simulated. The model used is based on Fig. (8). The model is general and is substantial for movement in three dimensional measurements. With the end goal of effortlessness, a planar case is considered here. The simulation is for a duration of 5sec. The first set of active and passive links make joint angles $\theta_{1A} = 60^\circ$ and $\theta_{1P} = -60^\circ$ with the horizontal axis. The second set of joint angles are $\theta_{2A} = 30^\circ$ and $\theta_{2P} = -30^\circ$. The third set of joint angles are $\theta_{3A} = 30^\circ$ and $\theta_{3P} = -30^\circ$. The standard Denavit-Hartenberg convention is followed.

The mechanism initially at rest hence all the velocities, translational and angular are zero initially. A sinusoidal torque profile is applied to each active joint simultaneously as given by

\[ \tau_{1Ain} = 0.001 \sin(\pi t) \]
\[ \tau_{2Ain} = 0.00025 \sin(\pi t) \]
\[ \tau_{3Ain} = 0.00005 \sin(\pi t) \]
Table 1: Initial parameters related to three joint prosthetic finger mechanism

<table>
<thead>
<tr>
<th>Properties</th>
<th>Active Links</th>
<th>Passive Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masses (kg)</td>
<td>( M_{1A} = 0.05 \text{ kg}; M_{2A} = 0.0375 \text{ kg}; ) ( M_{1P} = 0.05 \text{ kg}; M_{2P} = 0.0375 \text{ kg}; ) ( M_{3P} = 0.0281 \text{ kg} )</td>
<td>( M_{1A} = 0.0281 \text{ kg} )</td>
</tr>
<tr>
<td>Length (m)</td>
<td>( l_{x4} = 0.05 \text{ m}; l_{x2} = 0.0375 \text{ m}; ) ( l_{x3} = 0.0375 \text{ m} )</td>
<td>( l_{x1} = 0.05 \text{ m}; l_{x2} = 0.0375 \text{ m}; ) ( l_{x3} = 0.0375 \text{ m} )</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>( t_{yA1} = t_{yA2} = t_{yA3} = 0.01 ) ( t_{zA1} = t_{zA2} = t_{zA3} = 0.01 )</td>
<td>( t_{yP1} = t_{yP2} = t_{yP3} = 0.01 ) ( t_{zP1} = t_{zP2} = t_{zP3} = 0.01 )</td>
</tr>
<tr>
<td>EOMMI x-x</td>
<td>( I_{xxA} = m_i \left( \frac{i_{ix}^2 + i_{ix}^2}{12} \right) )</td>
<td>( I_{xxP} = m_i \left( \frac{i_{ix}^2 + i_{ix}^2}{12} \right) )</td>
</tr>
<tr>
<td>where ( i ) = 1,2,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EOMMI y-y</td>
<td>( I_{yyA} = m_i \left( \frac{i_{iy}^2 + i_{iy}^2}{12} \right) )</td>
<td>( I_{yyP} = m_i \left( \frac{i_{iy}^2 + i_{iy}^2}{12} \right) )</td>
</tr>
<tr>
<td>where ( i ) = 1,2,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EOMMI z-z</td>
<td>( I_{zzA} = m_i \left( \frac{i_{iz}^2 + i_{iz}^2}{12} \right) )</td>
<td>( I_{zzP} = m_i \left( \frac{i_{iz}^2 + i_{iz}^2}{12} \right) )</td>
</tr>
<tr>
<td>where ( i ) = 1,2,3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where; EOMMI = Element of mass moment of Inertia about

Table 2: Stiffness and damping values of various translational couplings used for three joint prosthetic finger mechanism

<table>
<thead>
<tr>
<th>Properties</th>
<th>Active Side</th>
<th>Passive Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness element ( K_i ) (N/m)</td>
<td>( K_{iA} = 10^4 \times [U] ) ( K_{iP} = 10^4 \times [U] )</td>
<td>( K_{iA} = 10^4 \times [U] ) ( K_{iP} = 10^4 \times [U] )</td>
</tr>
<tr>
<td>Stiffness element ( R_i ) (N-sec/m²)</td>
<td>( R_{iA} = 20 \times [U] ) ( R_{iP} = 20 \times [U] )</td>
<td>( R_{iA} = 20 \times [U] ) ( R_{iP} = 20 \times [U] )</td>
</tr>
</tbody>
</table>

Where; \( i = 1,2,3 \) and \([U]\) is a unit matrix.

Table 3: Values of the stiffness and damping of various rotational couplings

<table>
<thead>
<tr>
<th>Properties</th>
<th>Active Side</th>
<th>Passive Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness element ( k_{ix} ) (N/m)</td>
<td>( k_{ix} = k_{iy} = 1 \times 10^2 ) ( k_{ix} = k_{iy} = 1 \times 10^2 )</td>
<td>( k_{ix} = k_{iy} = 1 \times 10^2 ) ( k_{ix} = k_{iy} = 1 \times 10^2 )</td>
</tr>
<tr>
<td>Damping element ( r_{ix} ) (N-m/s²)</td>
<td>( R_{ix} = R_{iy} = 20 ) ( R_{ix} = R_{iy} = 20 )</td>
<td>( R_{ix} = R_{iy} = 20 ) ( R_{ix} = R_{iy} = 20 )</td>
</tr>
</tbody>
</table>
String-tube properties are $K_{x1} = K_{x2} = K_{x3} = 10^4 (N/m)$, and $R_{x1} = R_{x2} = R_{x3} = 20 (N·sec/m^2)$ for each joint pair. Radius of each joint pulley is 5 mm. There are two such pulleys on each joint.

### 3.1.2 Results

Fig. (9), (10) and (11) are shown below, the trajectories of center of mass of all links, extension in string and change in joints angle for active and passive side which describe that passive link follows same path as active side in opposite direction.

![Figure 9: Position of center of mass of all three links of active and passive side](image-url)

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Figure 10: Extension in string tubes

Figure 11: Angular trajectory of active and passive joints
3.1.3 Animation frames

4 Conclusion

Animation is a powerful tool for the understanding the pre-estimated motion of the mechanism which is also helpful for the verification of the simulated results. Animation is performed in the VRML environment which is an open source software. Interactive VRML animations, like those presented in this work, can be used in a variety of applications. VRML is a subset of the Open Inventor standard developed by SGI for their graphics workstation. VRML includes many of the things that go into making a world. It has a way of describing geometry which creates objects and spaces in which you can move around, as well as light, texture and sound which can be approached and viewed from whatever angle.
All these concepts and principles can be applied to any kind of mechanism for the visualization of 3D animation. With the help of this tool it now becomes easy to understand the simulation results. This work can be extended to model, simulate and animate the other simple as well as complicated mechanisms which are common in use and whose modelling, simulation and animation are difficult in other methodologies.

References