A Distributed Compliant Mechanism for a Piezo-actuated Flapping Wing
Nilanjan Chattaraj, G. K. Ananthasuresh, Ranjan Ganguli

Abstract
We present a distributed compliant mechanism, which acts like a transmission between a flapping wing of a micro air vehicle and a laminated piezoelectric actuator. The piezoelectric bimorph actuator is connected in the cantilever configuration with the compliant mechanism at its free tip. The mechanism takes translational deflection at its input from the piezoelectric actuator to provide angular displacement at its output, which causes flapping. We used topology optimization to obtain the design concept. The design of the mechanism is finalised using nonlinear elastic analysis. The final mechanism is a planar structure of 1 mm thickness and 40 mm $\times$ 24 mm in-plane footprint. The compliant mechanism exhibits 711 N/m input stiffness and 0.014 Nm/rad output torsional stiffness. The mechanism produces around 7° angular displacement per 1 mm input stroke, and around 8° angular displacement per 1 N force at its input. The mechanism has a fundamental frequency of 391 Hz, which is almost eight times greater than our assumed wing flapping frequency, which is 30 Hz. The final mechanism is prototyped with a 3D printer using VeroWhitePlus RGD835 material and tested with a piezoelectric bimorph actuator.

Keywords: Distributed compliant mechanism, Piezo-actuated flapping mechanism, flapping wing micro air vehicle

1 Introduction
Autonomous flapping-wing micro air vehicle (MAV) is one of the most challenging micro-engineering systems. Considering the complexity of modeling flapping wing aerodynamics, researchers adopted bio-mimicking approach for designing flapping wing MAV. Indirect insect flight mechanism being the simplest flight mechanism of all natural flyers is considered to be the best choice for bio-mimicking [1]. In indirect flight mechanism, the thorax contracts and expands to flap the wings. Tum-mala investigated electric motor-driven compliant mechanism for flapping wing [2]. Whereas, many researchers explored piezoelectrically actuated flapping-wing mechanism for its inherent advantages [3]. There are many approaches to implement piezoelectric actuator-driven flapping mechanism. Chung et al. [4], and Mukherjee and
Ganguli [5] investigated piezoelectric fan for flapping wings. This mechanism produces low flapping angle because of low deflection of the piezoelectric actuator. Sitti et al. [6], Khatait et al. [7] reported lumped compliant mechanism for flapping wings. In lumped compliant mechanisms, elastic deformation is concentrated in the flexural pivots, whereas in distributed compliant mechanisms, the deformation spreads throughout the whole structure. If the output is loaded, then a distributed compliant mechanism serves better than a lumped compliant mechanism. Therefore, piezoelectrically-actuated distributed compliant flapping mechanism finds its importance [8]. Stanford and Beran [9, 10] presented a conceptual design of piezoelectrically actuated compliant flapping mechanism using topology optimization. Their design consumes 1 N input stroke to produce only $1.6^\circ$ output angular displacement. We aim to improve this to achieve much larger flapping angle for the same force by adopting distributed compliant mechanism, co-designed with piezo-actuator.

We present a piezoelectrically actuated distributed compliant mechanism for flapping wings. The mechanism takes translational deflection from a laminated piezoelectric actuator at its input and generates angular displacement at its output to obtain flapping wings. A tapered piezoelectric bimorph cantilever actuator provides the input stroke to the compliant mechanism, and the mechanism at its output causes the wings flap against the aerodynamic load. Considering these input-output specifications we have also investigated the feasibility of the mechanism design using the spring-lever (SL) model [11]. The preliminary design of the mechanism is obtained by topology optimization method. The final design considers geometrical and structural nonlinearities and is prototyped using VeroWhitePlus RGD835 material with a 3D printer. Input and output stiffness of this compliant mechanism are 711 N/m and 0.014 Nm/rad. The compliant mechanism provides $7^\circ$/mm displacement transmission ratio and a resonant frequency of 391 Hz in its fundamental mode.

2 Modeling and Synthesis of a Compliant Mechanism

Figure 1a shows an approach to implement flapping by piezoelectrically actuated distributed compliant mechanism. We have considered a piezoelectric bimorph (Figure 1b) to drive the distributed compliant mechanism for this application. The mechanism...
flaps the wings against an aerodynamic load. The effect of that distributed aerodynamic load on each wing of length $L_w$ can be modeled as a resultant point force acting at the center of pressure on the wing. For a rectangular wing, the centre of pressure lies at $L_{cp} = \frac{3}{4}L_w$ distance from the wing joint. By considering the hovering condition for a MAV of mass $m$, the output moment on each wing is $M_{out} = \frac{mg}{2} L_{cp}$. A distributed compliant mechanism is designed to satisfy the input from the piezoelectric actuator and the output aerodynamic moment $M_{out}$.

### 2.1 Spring-lever Model of a Compliant Mechanism and a Feasibility Study

We use the spring-lever (SL) model [11] of a compliant mechanism to study the feasibility of the aforementioned specifications. The deformation and kinematics of a single-input single-output compliant mechanism can be expressed by a spring lever (SL) model. The SL model considers the displacement and force transmission of the compliant mechanism and provides insight about the feasibility of the design based on the input side and output side stiffness. Figure 2a shows a domain of compliant mechanism in piezoelectrically actuated flapping wing, and Figure 2b shows its equivalent SL model. Here, $F_a$ is the input force to the compliant mechanism exerted by a piezoelectric actuator, $k_a$ is the stiffness of the piezoelectric actuator, $x_{in}$ is the input displacement, $\Phi_{out}$ is the maximum output flap angle of the compliant mechanism, $k_{ci}$ is the input stiffness and $K_{co}$ is the output torsional stiffness for the compliant mechanism. The compliant mechanism flaps against the output moment $M_{out}$. Now by using static equilibrium conditions, we find the input stiffness of the compliant mechanism as

$$k_{ci} = \frac{F_a - k_a x_{in} - n M_{out}}{x_{in}} \quad (1)$$

and the output torsional stiffness of the compliant mechanism as

$$K_{co} = \frac{M_{out}}{\Phi_{out} - n x_{in}} \quad (2)$$
For a feasible design $k_{ci}$ and $K_{co}$ have to be always positive for the specified values of $k_a$, $n$, $x_{in}$, $\Phi_{out}$, $F_a$ and $M_{out}$. If $k_{ci}$ or $K_{co}$ is negative, then we can relax the input-output specification within the acceptable limits as follows

(i) input force : $F_{a|\text{min}} \leq F_a \leq F_{a|\text{max}}$
(ii) input displacement : $x_{in|\text{min}} \leq x_{in} \leq x_{in|\text{max}}$
(iii) output moment : $M_{out|\text{min}} \leq M_{out} \leq M_{out|\text{max}}$
(iv) output flap angle : $\Phi_{out|\text{min}} \leq \Phi_{out} \leq \Phi_{out|\text{max}}$
(v) actuator’s stiffness : $k_{a|\text{min}} \leq k_a \leq k_{a|\text{max}}$

to obtain positive values for $k_{ci}$ and $K_{co}$ to assure feasibility of the compliant mechanism design.

### 2.2 Synthesis of Compliant Mechanism by Topology Optimization

So far, we have accounted for the input-output specifications for the compliant mechanism and presented a technique to ensure the feasibility of the design. Now, we proceed to synthesize a distributed compliant flapping mechanism using topology optimization method. Topology optimization [12, 13, 14] is a numerical method to yield a structural solution from function level specifications. In topology optimization, we consider design region and discretize the region as in finite element framework. Then, we optimally distribute a given amount of material in that region. Finally, the topology is synthesized by selectively removing material from a discrete element by assigning some optimization criteria. Out of several different formulations of topology optimization, we have used flexibility-stiffness formulation [12], which is written as

\[
\begin{align*}
\text{minimize} & \quad J = -\frac{\text{MSE}}{\text{SE}} = -\frac{\int t_{cm} \sigma^T S \, d\Omega}{\int t_{cm} \sigma^T S \, d\Omega} \\
\text{subject to} : & \quad 0 \leq \rho_{cm}(x,y) \leq 1 \\
& \quad \int_{\Omega} t_{cm} \rho_{cm} \, d\Omega - V^* \leq 0 \\
& \quad \nabla \cdot \sigma + F_{in} = 0
\end{align*}
\]

Here, mutual strain energy (MSE) accounts for the output displacement and strain energy (SE) for the stiffness of the compliant mechanism. $\Omega$ indicates the 2D design domain, $t_{cm}$ the thickness and $V^*$ the specified volume of the compliant mechanism. Based on the requirements, the variable $\rho_{cm}$ varies from 0 to 1 at each discrete element. $\rho_{cm} = 1$ represents existence of the material at that discrete element and $\rho_{cm} = 0$ represents a hole. This variable is multiplied with the Young’s modulus of the material to give effective Young’s modulus $E = \rho_{cm}E_0$ of that discrete element. Here, $\mu$ is a penalty factor. Finally, the optimization problem (3) is solved to evaluate the value of $\rho_{cm}$ for each element, and we obtain the topology of the distributed compliant flapping mechanism.
3  Design, Results and Discussion

To design a piezoelectrically actuated distributed compliant mechanism for a flapping wing, we have considered BA4510 [15] piezoelectric bimorph cantilever actuator of 45 mm × 10 mm × 0.5 mm manufactured by PiezoDrive. The stiffness of this piezo-actuator is $k_a=200$ N/m, the mass 1.61 gm, and the fundamental resonant frequency 170 Hz. Here, we have considered 30 Hz flapping frequency. We consider 1 N block force for 500 V in parallel electrical connection, and output load on each wing $M_{out} = 0.74$ N.mm for hovering condition of a MAV of mass approximately 5 gm and wing length 50 mm. Input stroke $x_{in} = 1$ mm and amplitude of flap angle is varied between $\Phi_{out|\min} = 10^\circ$ to $\Phi_{out|\max} = 30^\circ$ (Table 1). Now, putting these data into the SL-

<table>
<thead>
<tr>
<th>Specification variables</th>
<th>Min</th>
<th>Max</th>
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<tr>
<td>$F_a$ (N)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x_{in}$ (mm)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$M_{out}$ (N.mm)</td>
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<td>0.74</td>
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<tr>
<td>$\Phi_{out} = \Phi_f$</td>
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<td>30$^\circ$</td>
</tr>
<tr>
<td>$k_a$ (N/m)</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
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model, we obtain the plot for input stiffness $k_{ci}$ vs. output torsional stiffness $K_{co}$ for different values of $n$ and $\Phi_{out}$. Figure 3 shows that the compliant mechanism design is feasible within the shaded region as $k_{ci}$ and $K_{co}$ both remain positive for specified input-output specifications. $n_{\max} = 0.524$ rad/mm for $\Phi_{out} = 30^\circ$, and $n_{\min} = 0.175$ rad/mm for $\Phi_{out} = 10^\circ$. Thus we ensure the feasibility of the compliant mechanism for the specified input-output parameters.

Now, to synthesize the topology of the compliant mechanism, we have considered one symmetric half of the mechanism, which is 20 mm × 20 mm. The domain is discretized into several square elements as shown in Figure 4a. A force is applied along the vertically upward direction at the bottom left corner of the design domain (Figure 4a), and an output displacement is expected along the vertically downward direction at the top right corner of the design domain as is shown in Figure 4a. Now, considering the equation (3) and performing the topology optimization [8, 16], we get a basic design of a distributed compliant mechanism for flapping wings (Figure 4b).

We finalized the topology by performing nonlinear elastic analysis. Then, we have considered properties of VeroWhitePlus RGD835 material to analyse the final design in Comsol® version 4.4. Young’s modulus of this material is 2.6 GPa and the density of this material is 1170 kg.m$^{-3}$. By finite element analysis we found the input stiffness of this mechanism as $k_{ci} = 711$ N/m and the output torsional stiffness as $K_{co} = 0.14$ N.m/rad. Maximum and minimum von Mises stress for $x_{in} = 1.5$ mm input stroke is 27 MPa and .38 mPa. Figure 5 shows the final mechanism, prototyped using VeroWhitePlus RGD835 material by a objet connex 3D printer. The deflection transmission factor $n = 7^\circ$/mm. Figure 6 plots the deflection $x_{in}$ caused by input force $F_{in} = 1$ N for variable frequency. The mechanism has 391 Hz resonant frequency in its fundamental mode. This is almost thirteen times greater than the 30 Hz flapping
Figure 3: Input stiffness $k_{ci}$ vs. output torsional stiffness $K_{co}$ for different values of $n$ and $\Phi = 10^\circ - 35^\circ$

Figure 4: (a) Design domain of one symmetric half of the compliant mechanism (b) synthesized topology of one symmetric half of the flapping mechanism

The frequency, which is considered here. Figure 8 shows a benchtop experimental setup to test the mechanism. A piezoelectric bimorph actuator, BA4510 (Figure 7), manufactured by PiezoDrive [15], is used for the experiment. We have used a voltage amplifier, PDu100B [17], to drive the actuator. The designed 3D compliant mechanism is tested with this actuator at $\pm 100$ V in parallel electrical connection, which produces $\pm 0.2$ N
Figure 5: 3D Prototype of the mechanism using VeroWhitePlus RGD835 material by a objet connex 3D printer

Figure 6: Frequency response and bandwidth of the distributed compliant flapping mechanism

block force with ±1 mm tip deflection.

4 Summary

We presented modeling and synthesis of a piezoelectrically actuated distributed compliant mechanism for flapping wings. We have quantitatively shown the feasibility of the design by using spring-lever (SL) model. Topology optimization technique is used to synthesize the topology of the design. Input, output stiffness and displacement transmission of the mechanism is calculated. The mechanism possess 391 Hz bandwidth, which is almost thirteen times more than the wing flapping frequency we considered. The final mechanism is 3D printed and the flapping is tested with a piezoelectric bimorph actuator.
Figure 7: BA4510 piezoelectric bimorph cantilever actuator of 45 mm×10 mm×.5 mm from PiezoDrive

Figure 8: Benchtop experimental setup for distributed compliant flapping mechanism driven by BA4510 piezoelectric actuator

References


